For Reference

NOT TO BE TAKEN FROM THIS ROOM

For Reference

NOT TO BE TAKEN FROM THIS ROOM

Ex libris
universitatis
albertaensis

UNIVERSITY OF ALBERTA
QUAECUMQUE VERA

Digitized by the Internet Archive
in 2018 with funding from
University of Alberta Libraries

https://archive.org/details/comparisonofcomp00muir

hesis
1960 (F)
# 32

THE UNIVERSITY OF ALBERTA

# A COMPARISON OF THE COMPETENCE IN ALGEBRA OF THE GRADE IX STUDENTS OF THE EDMONTON PUBLIC SCHOOLS IN 1938 AND 1959

A Thesis Submitted to
The Faculty of Graduate Studies
In Partial Fulfilment of
the Requirements for the Degree
MASTER OF EDUCATION

Division of Secondary Education

by
Norman Deans Muir
Edmonton, Alberta
September, 1960

# ABSTRACT

Much has been written in the past dozen years about the alleged deterioration of public education in Canada and the United States. Relatively little objective evidence has been brought forward, however, either in support or refutation of the claims of the critics; and of that which has been found, some is on one side, some on the other. The purpose of this study was to discover whether, in the subject of algebra, today's Grade IX students in the public schools of Edmonton are more or less proficient than were the students of 1938.

On June 12, 1959, a closely representative sample of 481 Grade IX students from six Edmonton junior high schools wrote the 1938 Grade IX Departmental Examination in Algebra. They also wrote the General Test (Ability Test) of the same year. Their mean raw scores on these tests were compared with those of a random sample of 203 Edmonton Grade IX students of 1938, whose record cards are on file at the Department of Education.

The 1959 students scored higher on the General Test (68 per cent) than did the 1938 students (63 per cent), but their mean algebra score was lower (36 per cent compared with 42 per cent). These differences in mean scores were found by statistical tests to be highly significant.

According to the available records, the mean I.Q. of the 1959 students was higher than that of the 1938

students, their year's attendance greater (188 days vs. 173 days), and the enrolment in their classes smaller (29 pupils vs. 37 pupils). Qualifications of teachers and pupil drop-out rates were comparable for the respective years covered by the study. Course contents were comparable on a topical basis, although the textbook used in 1959 contains only about half as much exercise material in algebra as does the textbook used in 1938.

The main conclusion of the study was that, according to the measuring instrument used, there has been a significant decline in competence in algebra at the Grade IX level in the Edmonton public schools since 1938. Some of the possible reasons for the decline which the study considers are: (1) a curricular policy of placing the chief emphasis on aspects of mathematics other than computational and manipulative proficiency; (2) a diminution in the exercise content of the course as revealed by a comparison of the former with the present textbook; (3) changes in social values and community attitudes which may have influenced the pupils' work habits and scholastic achievement.

# ACKNOWLEDGMENTS

The writer wishes to acknowledge with sincere thanks the assistance and co-operation he has received from many people during the preparation of this thesis. Thanks are extended to Mr. M. J. V. Downey, Director of Personnel and Research for the Edmonton Public School Board, for permission to carry out the necessary testing program in the schools, and to the principals and teachers who co-operated in the administering of the tests. The use of the files of the Department of Education and of the Edmonton public school system was essential to the progress of the study; and for free access to the required materials the writer is indebted to Mr. L. G. Frith and Mr. A. B. Evenson of the Department and to Mr. Downey and Mr. R. E. Shaul of the School Board office. Much help was provided by the secretaries and clerks at both of these places. In this connection, special acknowledgment is made of the cheerful and willing assistance given by Miss Iriye of the Examinations Branch, who not only knows where every record is kept, but seems to have a special sense as to what is likely to be most useful. Acknowledgment is made also of the helpful criticism and advice given by the following members of the Faculty of Education: Mr. D. H. Crawford, Mr. W. Pilkington, Dr. R. S. MacArthur, Dr. B. E. Walker and Mr. W. F. Coulson. Finally, recognition is given to the indulgence of the writer's wife and family, who for several hundred hours were relegated to secondary status by books, records, statistics and a typewriter.

## TABLE OF CONTENTS

CHAPTER PAGE

I. THE PROBLEM . . . . . . . . . . . . . . . . . 1

  The Problem . . . . . . . . . . . . . . . . 3

    Statement of the Problem . . . . . . . . . 3

    Definition of "competence" . . . . . . . . 3

    Importance of the study . . . . . . . . . 4

    Limitations of the study . . . . . . . . . 4

  Hypothesis . . . . . . . . . . . . . . . . 5

  Organization of the remainder of the thesis 6

II. REVIEW OF THE LITERATURE . . . . . . . . . . 8

  American Studies . . . . . . . . . . . . . 8

  Sansom's Five Year Study . . . . . . . . . 14

  Partlow's St. Catherines Study . . . . . . 17

  Commentary . . . . . . . . . . . . . . . . 25

III. BRIEF COMPARISON OF THE ALBERTA GRADE IX COURSES IN MATHEMATICS, 1938-1959 . . . . . . 28

  The Courses of Study . . . . . . . . . . . 28

    Curriculum guides . . . . . . . . . . . . 28

    Objectives . . . . . . . . . . . . . . . 29

  Textbooks . . . . . . . . . . . . . . . . . 31

  Commentary . . . . . . . . . . . . . . . . 40

IV. INITIAL PROCEDURE, THE TESTS, AND THE SAMPLES 44

  Initial Procedure . . . . . . . . . . . . . 44

  The Tests . . . . . . . . . . . . . . . . . 45

CHAPTER PAGE

The Algebra Test . . . . . . . . . . . . 46
Analysis of the Algebra Test . . . . . . . 47
The General Test . . . . . . . . . . . . 51
The Samples . . . . . . . . . . . . . . . 51
The 1938 Sample . . . . . . . . . . . . . 51
The 1959 Sample . . . . . . . . . . . . . 52
Representativeness of the sample . . . . 56
V. SCORING AND TEST RESULTS . . . . . . . . . . 63
The Algebra Test . . . . . . . . . . . . 63
The General Test . . . . . . . . . . . . 65
Summary of Results . . . . . . . . . . . 66
VI. STATISTICAL ANALYSIS OF DATA; TESTS OF SIGNIFICANCE . . . 73
Statistical Theory . . . . . . . . . . . 73
Significance Tests . . . . . . . . . . . 76
The Algebra Test . . . . . . . . . . . . 76
The General Test . . . . . . . . . . . . 84
Conclusion . . . . . . . . . . . . . . . 85
VII. FACTORS AFFECTING THE RESULTS . . . . . . . . 87
Measurable Factors . . . . . . . . . . . 87
Factors not subject to precise quantitative assessment 90
Summary . . . . . . . . . . . . . . . . . 100
VIII. SUMMARY AND CONCLUSIONS . . . . . . . . . . . 102
Summary . . . . . . . . . . . . . . . . . 102

CHAPTER PAGE

Conclusions . . . . . . . . . . . . . . . . . . 104

Interpretation . . . . . . . . . . . . . . . . 106

Finale . . . . . . . . . . . . . . . . . . . . 117

BIBLIOGRAPHY . . . . . . . . . . . . . . . . . . . 120

APPENDIX A. Table from the article "Arithmetic Then and Now," by Mabel E. Boss . 125

APPENDIX B. Letter requesting the co-operation of the principals of the selected schools . . . 126

APPENDIX C. Instructions to the supervising teacher . . 129

APPENDIX D. The Algebra Test . . . . . . . . . . 132

APPENDIX E. Scoring key for the Algebra Test . . 139

APPENDIX F. The General Test . . . . . . . . . . 144

APPENDIX G. Statistical Theory . . . . . . . . . 152

APPENDIX H. Statistical Calculations . . . . . . 158

## LIST OF TABLES

TABLE PAGE

I. Comparison of the Grade IX Mathematics Textbooks used in Alberta in 1938 and 1959 . . 32

II. Analysis of the 1938 Grade IX Departmental Examination in Algebra . . . . . . . . . . 50

III. Distribution of Scores on the Grade IX Ability Test for the 1959 Population and Sample . . 59

IV. Mean Raw Scores and Percentages on the Algebra Test and the General Test for the 1959 and 1938 Samples . . . . . . . . . . . . . . 66

V. The Number of Marks Obtained on Each Question of the 1938 Grade IX Departmental Examination in Algebra by a Sample of 481 Edmonton Grade IX Students, June, 1959 . . . . . . . . . 71

VI. Sample Size, Mean Raw Score, Standard Deviation and Variance for the Algebra Test, 1938 and 1959 . . . . . . . . . . . . . . 80

VII. Sample Size, Mean Raw Score, Standard Deviation and Variance for the General Test, 1938 and 1959 . . . . . . . . . . . . . . 84

VIII. Factors, Measurable in Some Degree, Which Would Be Expected to Influence the Test Scores of the Two Samples . . . . . . . . . 88

# LIST OF FIGURES

FIGURE PAGE

1. Sketch Map of the City of Edmonton showing the Locations of the Six Schools Used in This Study . . . . . . . . . . . . . . . . 57
2. Frequency Polygon Showing the Distribution of Scores on the Grade IX Ability Test for the 1959 Population . . . . . . . . . . . . . . 61
3. Frequency Polygon Showing the Distribution of Scores on the Grade IX Ability Test for the 1959 Sample . . . . . . . . . . . . . . . . 62
4. Frequency Polygons Showing the Distribution of Scores on the General Test for the 1959 and 1938 Samples . . . . . . . . . . . . . 68
5. Frequency Polygons Showing the Distribution of Scores on the Algebra Test for the 1959 and 1938 Samples . . . . . . . . . . . . . 69
6. Frequency Polygons Showing the Distribution of Scores on the General Test and the Algebra Test for the 1959 Sample . . . . . . . 70

# A COMPARISON OF THE COMPETENCE IN ALGEBRA OF THE GRADE IX STUDENTS OF THE EDMONTON PUBLIC SCHOOLS IN 1938 AND 1959

# CHAPTER I

# THE PROBLEM

For more than ten years public education in Canada and the United States has been the object of a critical attack of unprecedented concentration. The critics have declared in books, magazine articles, newspaper columns, broadcasts, speeches, resolutions and interviews[1] that educational standards have declined, that curricula have become cluttered with "frills", that teaching methods have become ineffective, that courses have been "watered down", that "social adjustment" has been substituted for learning, that "progressivism" has eaten like a rot into the foundations of education, and that the schools have been turning out a substandard product, deficient in the skills of reading, writing, spelling and calculating. Most of the critics have been laymen (in the strict sense), some of them closely associated with higher education. Educationists, on the other hand, have been in the main defenders of the techniques they practise and champions of the underlying philosophies; they have maintained that the adverse criticisms have been subject-

---

[1] See the bibliography, page 120.

ive and emotional, founded on misunderstanding and incomplete evidence. These opposing factions have been so insistent and vocal that governments have been forced to pay attention to them, with the result that Royal Commissions have been appointed in Ontario, Alberta and Manitoba to investigate nearly every aspect of public education in their respective provinces. These Royal Commissions have recently completed the most comprehensive surveys of education ever to be published in this country.[2]

Yet little has been done (in Alberta at least) by way of controlled, experimental studies to discover whether standards have indeed fallen, or have been maintained, or improved. The present investigation is an attempt to provide evidence, by means of sampling and testing, which may shed some light on this unsettled and unsettling question.

---

[2] J. A. Hope *et al.*, *The Report of the Royal Commission on Education in Ontario* (Toronto: Baptist Johnston, Printer to the King's Most Excellent Majesty, 1950); D. A. Cameron *et al.*, *The Report of the Royal Commission on Education in Alberta* (Edmonton: L. S. Wall, Printer to the Queen's Most Excellent Majesty, 1959); R. O. MacFarlane *et al.*, *The Report of the Manitoba Royal Commission on Education* (Winnipeg: Queen's Printer, 1959).

## I. THE PROBLEM

Statement of the problem. It is the purpose of this study: (1) to compare the standard of competence in Algebra of the Grade IX students of the Edmonton public schools in 1959 with that of their counterparts of 1938; (2) to discover whether the difference in competence, if there is a difference, is statistically significant; (3) to discuss some of the factors which may have contributed to a change in competence.

Definition of "competence". The ordinary meaning of competence is proficiency, or expertness, or the ability to perform a useful skill correctly and effectively. The emphasis is on performance. Performance, however, is applied principle. Competence is therefore defined for the purposes of this study as an understanding of the principles and a high degree of mastery of the techniques of a particular area of study (in this case, Algebra).

In the present inquiry, the measure of a pupil's competence is his score on the 1938 Grade IX Departmental Examination in Algebra.

Importance of the study. No worthwhile answer can be given to the question, "Have standards changed?" until some sort of reliable measurement is made. The level of achievement in a given subject area of a sample of today's students must be measured as objectively as possible against the level of achievement of a like sample of students from a selected past year in the same subject area. With the results of such a comparison at hand, one may be permitted a judgment as to whether today's students in a given grade, performing in the same subject area, appear to be better or less well equipped than their counterparts of the chosen former year. To the present writer's knowledge, no such investigation has been done in Alberta. It is submitted that the present study, in spite of its shortcomings, constitutes a more objective basis for a long-term comparison of past and present standards in a specific subject area than has yet been provided in this province.

Limitations of the study. Three limitations have been imposed on this study. First, the subject matter area has been confined to Algebra. It was found impossible to make a wider investigation of mathematical abilities because the course contents of the years selected for comparison were too different with respect to

the sections on Geometry. Second, the populations compared are those of the Edmonton public schools only. The city separate schools and districts in other parts of the province were excluded so that the task would not be excessively cumbersome. It is not assumed, however, that the results would have been the same if other systems had been included. Third, the samples compared were taken from the Grade IX populations of 1938 and 1959. The year 1938 was chosen because it was approximately a generation removed from the year of the study, and because the students of that year had received their elementary schooling in what might be termed the "pre-progressive" era. The year 1959 was chosen for the obvious reason that it was the current year and its students were available for testing.

*Hypothesis*. The hypothesis that this study proposes to test is that *there is no difference in competence in Algebra between the Grade IX students of 1959 and those of 1938 in the Edmonton public school system*, according to the measuring instrument employed. This measuring instrument was the 1938 Grade IX Departmental Examination in Algebra.

Organization of the remainder of the thesis. The thesis proper begins in Chapter II with a review of the literature of comparative studies of a nature similar to the present one. First, some American studies are discussed briefly. Following this is a summary of the pertinent portions of the five-year investigation carried out in Alberta by Sansom from 1944 to 1950.[3] Finally, there is a digest of the major Canadian experiment of this type, Partlow's Arithmetic and Reading, Yesterday and Today.[4]

These reviews are followed in Chapter III by a comparison of the Alberta Grade IX mathematics curricula and textbooks for 1938 and 1959. Chapter IV outlines the initial procedures of the present study, discusses the tests which were used, and describes the samples and the methods of selecting them. Scoring of the tests and a summary of results are dealt with in Chapter V. The statistical analysis of the test results occupies Chap-

---

[3] Clarence E. Sansom, "Statistical Reports, Grade X Survey Tests." The A.T.A. Magazine, July, 1944; November, 1945; September-October, 1947; June, 1948; June, 1949; June, 1950. A summary appears in the June, 1950 issue, pages 28 to 33.

[4] Hugh R. Partlow, Arithmetic and Reading, Yesterday and Today (Toronto: The Copp Clarke Co., Limited, 1955).

ter VI; and this is supplemented in Chapter VII by a discussion of factors which may have had differential effects on the achievements of the students comprising the two samples. Chapter VIII presents a summary and conclusions. Appendices contain relevant material not included in the text.

# CHAPTER II

# REVIEW OF THE LITERATURE

## I. AMERICAN STUDIES

### The St. Louis Study Reported by Boss.

A study by the division of tests and measurements of the St. Louis (Missouri) public school system and reported by Boss,[5] although covering an earlier period, is of interest for its similarity to the present investigation and for the results of its findings. In May, 1938, *The Cleveland Survey Arithmetic Test*, *Form I*, was administered to a sample of 919 pupils in all grades above Grade II in the St. Louis public schools. This test had previously been used in St. Louis by Judd in 1916 at the same grade levels, and the results of the earlier testing were available for comparison. Judd's sample had been random; the 1938 sample was selected for similarity to the city medians and distributions after these had been established by the preliminary use of standardized achievement tests.[6]

---

[5] Mabel E. Boss, "Arithmetic Then and Now." *School and Society*, March 23, 1940, p. 391.

[6] *The Modern School Achievement Test* and the *Pressey Third Grade Attainment Scale*.

The results showed that throughout the grades the median scores of the 1938 pupils were lower than those of the 1916 pupils. Comparative scores for the fourth quarters of Grades III, VI and VIII are shown in Table I of Boss's article (see Appendix A, p.125 ).

Boss gives two distinct reasons to account for the lower achievement in 1938. The first is that between 1916 and 1938 there were at least three simplifications in the arithmetic curriculum, all designed to put forward to higher grades the presentation of certain subject matter. The second and main reason is that the 1938 pupils made up a less select group, owing to a lower drop-out rate. Figures are provided to support this contention. For instance, the Grade VIII pupils in 1916 made up 4721/72160, or 6.5 per cent, of the total enrolment; whereas in 1938 the Grade VIII classes contained 6432/60798, or 10.6 per cent, of the enrolment. Moreover, in 1916, 72 per cent of the Grade VIII pupils were 14 years old or older; in 1938 only 53 per cent were in this age group. Boss cites a St. Louis research report of 1935 to show that the elimination of pupils from the elementary grades before graduation is qualitative. Thus in 1916 the larger drop-out rate removed the less apt scholastically, leaving the pupils of higher

ability in school. The increased enrolment of 1938, according to Boss, "consists largely of those pupils who would have dropped out in 1916 because they were not successful in school."[7] She mentions also that the distribution of the 1938 test scores shows that " . . . the upper range is similar to that of 1916 and equally high."[8] Boss sums up the experiment thus:

> The conclusion to be drawn from the investigation is that the lower scores of 1938 do not indicate lower efficiency in arithmetic, but rather that the curriculum has been deliberately simplified and that the pupils are younger and less selected than those of 1916. Not the lowering of standards as is often claimed, but the widening of opportunity for continued school attendance, produces the lower mass average. Individual achievement remains as high as it formerly was for selected pupils; but group achievement has been lowered because the group includes "all the children of all the people" in keeping with the democratic purposes of modern education.[9]

## A Summary of Studies Prepared by The National Education Association

A brief mimeographed summary of a number of related research studies has been published under the title, The Three Rs Hold Their Own at the Midcentury, by The National

---

[7] *Op. cit.*, p. 392.

[8] *Op. cit.*, p. 392.

[9] *Op. cit.*, p. 392.

Education Association of the United States.[10] The initiation of this compendium is described in the opening paragraph of the Introduction:

> Continuing requests for answers to unfavorable criticism about present-day teaching of fundamentals, or basic skills, prompted the NEA Research Division to write to directors of research in city school systems for "grass roots" evidence. This attempt in July 1950 to assemble local surveys and studies, articles, and other unpublished materials . . . brought replies from about 55 of the 440 directors asked for information. Approximately half of the replies contained objective evidence of pupil achievement in the local school systems. The present memorandum reports the available results of standardized tests conducted in city schools during the past 30 years . . .[11]

The memorandum consists largely of tables of scores on various standardized tests in arithmetic, reading, spelling, general achievement, etc. Many of the figures compare achievement levels in certain school systems with the national norms over short periods of time. Some, however, make long-period comparisons. In Florida, for example, the Stanford Arithmetic Test given to Grade VI pupils in 1927 and again in 1949 indicated an increase

---

[10] The Three Rs Hold Their Own at the Midcentury. A summary of research studies prepared with the assistance of directors of research in city school systems. Research Division, National Education Association of the United States. Washington, D. C., 1951.

[11] Ibid., p. 3.

in median grade level from 5.9 to 7.0. In Dearborn, Michigan, the use of the same test in 1928 and the new form of the test in 1948 showed that the achievement of Grade VI pupils rose from a median of 5.8 to a median of 7.2. Similar testing in Dayton, Ohio, showed an increase in achievement level; in this case the medians were for the most part below the national norms. A twelve-year testing program (1939-1950) in Kalamazoo, Michigan, indicated that the educational quotients of Grade VI-A pupils have in general been equal to or slightly higher than the intelligence quotients.

Sixth grade tests in arithmetic given to pupils in Evanston, Illinois, in 1928 (Stanford) and in 1950 (Progressive) showed that the percentages of both above-average and below-average scores were smaller at the end of the period, and the percentage of average scores was larger. Another testing program in Illinois revealed that, although the level of intelligence of the 616 elementary school pupils tested in 1949 was lower than that of 3900 pupils measured by the same test in 1930-33, the pupils' scores on standardized tests in reading and arithmetic were as good as in 1930.

Results of a different character were obtained

from the administration of the New York Test on Mixed Fundamentals (arithmetic) to 1135 tenth grade students in New York city, December, 1949. Forty-six per cent of those tested rated sixth grade achievement or lower, seventy-two per cent showed less than ninth grade ability, and only twenty-three and one-half per cent performed at the tenth grade level or higher. Tests were conducted in New York city in 1940-41 to discover any change in arithmetic competence which might have occurred since the introduction of an experimental activity-type program in 1935. The results indicated differences so slight as to be statistically non-significant.

The general conclusion of the summary is that on the whole achievement levels have been maintained or have increased. The memorandum states that " . . . while no definite and consistent trend is discernible, and results are not entirely comparable, the general impression derived from these reports is that present-day pupils for the most part equal, and often excel, the achievement of pupils in similar grades in the past."[12]

---

[12] Ibid., p. 5.

## II. SANSOM'S FIVE YEAR STUDY

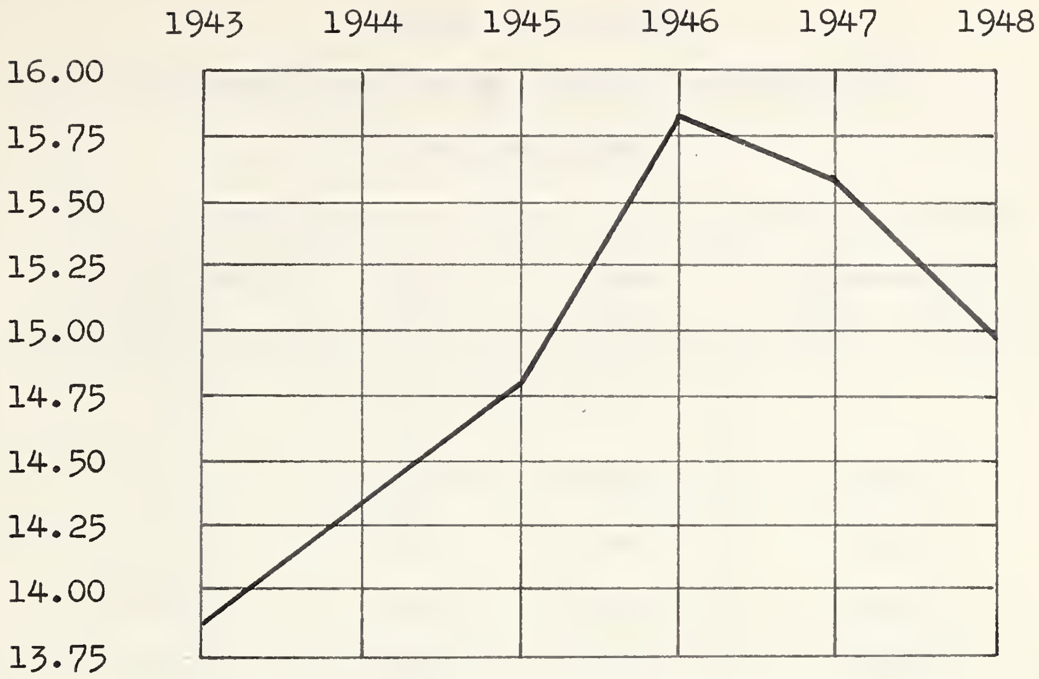
The five year study of school achievement at the Grade X level made in Alberta for the Alberta Teachers' Association by Sansom[13] between 1944 and 1950 attracted wide attention in this province. Sansom's purpose was "to get objective evidence on the much discussed question as to whether the grounding of high school pupils in the lower grades preparatory to entering high school is improving or declining."[14] His plan was to give identical tests in Vocabulary, Mathematics, English Usage, Science and Social Studies for at least five years and to examine the results for indication of trends. The tests were specially prepared for the study, and were given to about 2,000 Grade X students each year. The following graph and table are taken from the summary of the study which appeared in the June, 1950 issue of The A.T.A. Magazine.

The trend of the mean scores in mathematics for the five year period is shown graphically:

---

[13] Clarence E. Sansom, "Statistical Reports, Grade X Survey Tests." The A.T.A. Magazine, July, 1944; November, 1945; September-October, 1947; June, 1948; June, 1949; June, 1950. The last issue contains a summary, pp. 28-33.

[14] Ibid., p. 28.



TREND OF MEAN SCORES IN MATHEMATICS OF APPROXIMATELY 2,000 GRADE X PUPILS, 1943-48 (SANSOM). FROM THE A.T.A. MAGAZINE, JUNE, 1950, PAGE 32

Sansom's Table 12, page 16, gives a percentile ranking of scores for the first and last years of the study. It shows consistently higher scores for 1948.

With respect to the other subject areas of Sansom's study, there was a general improvement in English Usage and Science, a small decline in Vocabulary, and a more pronounced and steady decline in Social Studies.

### TABLE 12 (SANSOM)

### PERCENTILES, TOTAL SCORES[α] (MATHEMATICS)

### 1943 and 1948

| Percentile Points | 1943 (2194) | 1948 (1842) | Difference 1948 - 1943 |
|---|---|---|---|
| P 100 | 41.50 | 41.50 | |
| P 97 | 29.52 | 31.10 | +1.58 |
| P 90 | 23.80 | 25.42 | +1.62 |
| P 80 | 19.54 | 21.43 | +1.89 |
| P 75 | 17.97 | 19.89 | +2.08 |
| P 70 | 16.74 | 18.53 | +2.21 |
| P 60 | 14.59 | 16.15 | +1.56 |
| P 50 | 12.81 | 14.04 | +1.23 |
| P 40 | 11.02 | 12.09 | +1.07 |
| P 30 | 9.42 | 10.22 | +0.80 |
| P 25 | 8.62 | 9.30 | +0.68 |
| P 20 | 7.76 | 8.35 | +0.59 |
| P 10 | 5.84 | 5.92 | +0.08 |
| P 3 | 3.28 | 3.32 | +0.04 |
| P 0 | 0.00 | 0.00 | |

---

[α] In a previous table Sansom displays the percentile scores of boys and girls separately. By "total scores" here he means the combined scores of boys and girls.

Sansom's principal conclusions were that: (1) the *steadiness* of Alberta's elementary education in the disturbed war and post-war period was impressive; (2) the general tendency in achievement was upward; (3) most of the noticeable changes were small; (4) there was *no suggestion of a general decline* (italics in the original).

## III. PARTLOW'S ST. CATHERINES STUDY

The major Canadian study in which standards of pupil achievement are compared over a considerable period of time is that carried out by Partlow in St. Catherines, Ontario, from 1952 to 1954. Partlow's study was the subject of a doctoral dissertation, but it has been published under the title *Arithmetic and Reading, Yesterday and Today*.[15] The purpose of the study was "to compare the Arithmetic and Reading standards in 1952-54 with those of 1933-38 by using the same objective tests, to summarize significant educational changes during that period which might affect standards, and finally, to discover by means

---

[15] Hugh R. Partlow, *Arithmetic and Reading, Yesterday and Today*: A Comparison of St. Catherines Public School Standards, 1933-38 and 1952-54 (Toronto: The Copp Clarke Co., Limited, 1955).

of the available data, strengths or weaknesses in the New Programme which was used from 1938 to 1954."[16] The scope of the study was quite wide: it made comparisons of standards for all St. Catherines pupils in Grades V, VI, VII and VIII in arithmetic and reading (including vocabulary). Altogether, the test scores of 3,295 pupils who wrote at the beginning of the period were compared with the scores of 3,499 pupils who wrote the same tests at the end of the period.

Only that part of Partlow's study dealing with arithmetic will be reviewed here.

All of the arithmetic tests used were objective and standardized, and all were published in Ontario. _Form A, Dominion Arithmetic Test of Fundamental Operations_, Grades IV-VIII, 1934 edition, was given to all regular Junior Third, Senior Third and Junior Fourth public school pupils in St. Catherines on or about March 1, 1936, and to all Grade V, VI and VII public school pupils on March 1 or 2, 1954. _The Dominion Group Achievement Test_, Part II, Arithmetic Computation and Reasoning, Grade VIII, 1934 edition, Form A, was given to all St. Catherines Grade VIII pupils on June 5, 1952. Form D of the same test was

---

[16] _Ibid._, p. 3.

given to all Grade VIII pupils on June 5, 1953. The 1934 norms were available for comparison.

Partlow takes two lengthy chapters to compare conditions in the St. Catherines school system at the beginning of the period with those at the end, and to discuss developments which took place in the interval. The following are the main factors he considers:

1. General administration and supervision. No marked changes in policy or practice are noted.
2. Supervision within the schools. More time was allowed for supervision by principals in the fifties than in the thirties.
3. Teacher-pupil ratio improved during the period. Teacher-class ratio in 1936 was 1.04; in 1954, 1.08.
4. Enrolment per classroom had decreased. The average (taken November 30) for 1934-36 was 40.4; for 1952-54, 36.0.
5. Promotion and non-promotion. The average failure rate in 1934-36 from the Kindergarten to Grade VIII was 18.2 per cent. In 1952-54 it was 6.1 per cent. Partlow says that changes in promotion policy during the period made the Grade VIII population of the fifties a less select group than that of the thirties.

6. Drop-outs. In the fifties, 20 per cent more pupils remained in school and were promoted from Grade VIII to Grade IX than in the thirties.

7. Attendance. Pupils in 1951-52 attended on the average three and one half days more during the school year than in 1933-36.

8. The average age of pupils at most grade levels was higher in the fifties than in the thirties. In Grade VIII the difference was nearly a year.

9. Over-age and under-age pupils. Classes were more homogeneous with respect to age in the fifties than in the thirties.

10. Intelligence. No significant change in mean I.Q. was revealed by the available figures.

11. Reading habits. There was a decline in per capita public library circulation for both adults' and children's books from 1933 to 1953.

12. Socio-economic changes. The increase in population from the thirties to the fifties brought into St. Catherines nearly twice as many pupils in the below-average socio-economic bracket as in the above-average category.

13. New Canadians. The forty-four New Canadians in the St. Catherines schools in 1953-54 had no effect on the arithmetic scores one way or the other, according

to Partlow. (There was an effect on the vocabulary test).

14. Home and School Associations. In 1933 there was one Home and School Association in St. Catherines, with about fifty members. In 1954 there were ten Associations with a total of 1478 members. Partlow says, "Home and School influence upon the education of the boys and girls has been great."[17] He believes that this growth has had a positive effect on the arithmetic and reading achievement in the public schools.

15. Qualifications of teachers. In the period from 1933 to 1954 the percentage of teachers holding First Class certificates rose from 26 per cent to 83 per cent. The number of Second Class certificate holders declined from 70 per cent to 6 per cent in the same period. The number of special certificates and degrees rose from 6 per cent to 90 per cent.

16. Teacher preparation. There was a general improvement in in-service training and in the professional status of teachers.

17. Changes in the philosophy of teaching. In this area the emphasis shifted from rather rigid requirements with respect to knowledge and examination results to an aim on the part of the school to give to each

---

[17] *Ibid.*, p. 59.

child the training that he needed "to develop him as a healthy, happy and useful individual."[18] The activity program was stressed. In the teaching of arithmetic, emphasis shifted from drill intended to give the pupil speed and accuracy in computation to stimulating the child's interest in number and to making problems more meaningful. Diagnosis of difficulties and remedial measures received more stress. The introduction of some topics was postponed in the revised course; other topics were introduced earlier. More freedom was allowed in the pattern and methods of solution of problems.

18. Textbooks underwent a continual improvement.

## Arithmetic Data and Analysis

Partlow considers the Dominion Arithmetic Test of Fundamental Operations, Grades IV to VIII, 1934 edition, the most important of the tests. He found that the 1954 scores showed a significant improvement in percentage of questions correct over the 1936 scores at the same grade levels. The gain at the Grade V level was 4.2 per cent; at the Grade VI level, 15.5 per cent; at the Grade VII level, 6.9 per cent. An item analysis revealed that there was a slight decline in proficiency in the sub-

---

[18] Ibid., p. 74.

traction, multiplication and division of integers; but there was an improvement in seven other areas, namely, weights and measures, addition and subtraction of fractions, multiplication and division of fractions, addition and subtraction of decimals, multiplication of decimals, percentage, and percentage and decimals. The improvement in the addition and subtraction of fractions was nearly 30 per cent. The general improvement was 7.2 per cent.

The most notable improvement was at the Grade VI level; here, the 1954 mean scores were higher in every operation except multiplication, which showed a decline of only 1.1 per cent. The increase in the Grade VI median score was from 26 to 36. Partlow points out that:

> A summary of all items . . . reveals that 9.2 per cent of the items had the same percentage of correct responses at the end of the period, whereas 25.4 per cent showed a decline, and 65.4 per cent showed an improvement.
>
> The fact that 157 of the 240 items, or more than 65 per cent, showed an improvement indicates that the improvement was not confined to a few types of questions but was quite general.[19]

On the Dominion Test in Arithmetic Computation

---

[19] Ibid., p. 97.

*and Arithmetic Reasoning* for Grade VIII, Forms A and B, administered in 1952 and 1953 respectively, there was an improvement of 5.05 per cent in the mean score over the 1934 Ontario norms.

Partlow sums up the percentage gains for the various sub-tests as follows:[20]

| | | |
|---|---|---|
| 1. | Arithmetic Fundamentals, Grade V | 4.2% |
| 2. | Arithmetic Fundamentals, Grade VI | 15.5% |
| 3. | Arithmetic Fundamentals, Grade VII | 6.9% |
| 4. | Arithmetic Computation (A) Grade VIII | 4.8% |
| 5. | Arithmetic Computation (B) Grade VIII | 6.4% |
| 6. | Arithmetic Reasoning, Form A, Grade VIII | 6.5% |
| 7. | Arithmetic Reasoning, Form B, Grade VIII | 2.0% |

### Statistical Analysis of the Scores

Using standard methods of statistical analysis (chiefly the t-test of significance for differences in the means of the class medians, and the Chi-square test of independence), Partlow found that the improvements in the scores on the *Dominion Arithmetic Test on Fundamental Operations* were significant at the Grade VII level and highly significant at the Grade V and VI levels. He found also that the improvement over the 1934 Ontario

---

[20] *Ibid.*, p. 99.

norms on the Dominion Test in Arithmetic Computation and Arithmetic Reasoning at the Grade VIII level was highly significant. In summing up the statistical analysis, he expresses his belief that the same kind of results would be revealed by similar studies in other Ontario cities.

## Commentary

The conclusions of the American studies reviewed in this chapter are not, in the writer's opinion, completely satisfactory. Boss's case for the present (i.e., 1938) seems to be that standards have not really declined; rather, the curriculum has been simplified and the students are less bright, on the average, than formerly. The present standards are not lower, they are merely different; and conditions are better this way because they ensure a more uniform education for all the children of all the people. So Boss argues. But many critics have claimed that this philosophy has resulted in a neglect of the needs of the brighter pupils, and in fact steps are being taken in some school systems to correct this situation. Behind these arguments is the philosophical question: What kinds of standards are best in education, and what kinds of services should the schools offer?

A legitimate question to direct at Boss is this: Granted that the school population is less able today than it formerly was, cannot the public expect this decline to be offset by a general improvement in the efficiency of the educational system? If not, what is the use of today's elaborate facilities, special services to ensure regular attendance, better trained teachers, and continuing research to develop more effective teaching methods?

A notable feature of the St. Louis study and of those summarized by the NEA is that they were internally conducted in the various school systems. This admits the possibility of unconscious bias, especially in the interpretation of results. Different conclusions may have been reached by independent researchers. Moreover, only about 28 of the 440 school systems contacted by the NEA, or about 6½ per cent, submitted objective evidence of pupil achievement. Possibly some who had conditions like those reported from New York preferred to remain silent. In other words, the NEA sample may have been an extreme deviate.

The studies by Sansom and Partlow appear to be quite reliable. The methods were objective, and there

is no question of explaining away an undesirable result.

In view of the wide and critical notice that has been taken of elementary and secondary education in the last ten or fifteen years, the extension and continuation of this kind of research would serve a valuable purpose.

# CHAPTER III

# A BRIEF COMPARISON OF THE ALBERTA GRADE IX COURSES IN MATHEMATICS, 1938-1959

## The Courses of Study

*Curriculum guides*. There is a considerable contrast between the curriculum guides for Grade IX Mathematics issued by the Department of Education for use in 1938 and in 1959. The Department's directives and suggestions to mathematics teachers in the Intermediate School of 1938 are found in the short space of four pages of the 280-page *Programme of Studies for The Intermediate School*.[21] Of this material, slightly more than one page is devoted to the Grade IX course; and of this page, a single paragraph of five lines contains the official statement of the purpose and intent of the section on algebra.

In 1959, however, the Department supplied a sep-

---

[21] *Programme of Studies for The Intermediate School and Departmental Regulations Relating to the Grade IX Examination* (Edmonton: A. Shnitka, King's Printer, 1935).

arate 44-page booklet[22] which sets forth in considerable detail the objectives and basic principles of organization of the junior high school course in mathematics, and discusses in a specific manner a number of techniques of presentation. In the possession of this explicit guide the teacher of 1959 (especially the inexperienced teacher) had a decided advantage over the teacher of 1938. Chapter IV, comprising pages 31 to 44 of the guide, deals with the Grade IX course, and makes clear to the teacher the official Departmental position with respect to aims and acceptable techniques of presentation in the teaching of mathematics at this level. It concludes with a comprehensive mathematics word list, arranged to correspond with the chapters of the textbook.

Objectives. There appears to have been little change in the objectives of the Grade IX mathematics course from 1938 to 1959. The Programme of Studies for 1938 has this to say:

> One may assume that the majority of children in Alberta will continue their schooling at least to the end of the Intermediate School. It follows that the mathematics of Grades VII, VIII and IX must be

---

[22] Junior High School Curriculum Guide for Mathematics (Province of Alberta, Department of Education, 1952).

> of interest to all pupils. The content must be suited to the general-course pupil as well as to the future matriculant. It must, to be understood, be related to the child's out-of-school environment. It should, wherever possible, include activities and references to concrete situations.
>
> Many pupils will not continue their schooling beyond Grade IX. It is not desirable, therefore, to spend their time on perfecting techniques of computation which they will seldom if ever use. It is preferable to introduce them to as wide a range of interpretative mathematics as possible . . .
>
> _Since the aim of the course is to interpret the environment rather than to give skill in computation, emphasis is placed on easy mathematical relationships expressed in the form of tables, graphs, formulas and equations._[23]

The _Curriculum Guide_ for 1959 lists these objectives for the junior high school course in mathematics:

> 1. To develop the ability to think clearly;
> 2. To develop the ability to use information, concepts and general principles;
> 3. To develop the ability to use fundamental skills;
> 4. To develop desirable attitudes, especially respect for knowledge, respect for good workmanship, respect for understandings, social mindedness, open mindedness;
> 5. To develop a wide range of interest and appreciation.[24]

---

[23] _Op. cit._, p. 120.

[24] _Op. cit._, p. 3.

And in the preamble to Chapter V (The Grade IX Course) appears the statement:

> Our aim should be to have the pupils develop the faculty of clear thinking, skill in using formulas and solving problems so that they will be able to appreciate the maze of scientific development which stems from an ever increasing body of mathematics.[25]

Textbooks

A comparison of the textbooks used at the beginning and end, respectively, of the period covered by this study is given in Table I on the following pages. The book used in 1938 was *Mathematics for Today*, by LaZerte and Betz;[26] that used in 1959 was *Mathematics for Canadians*, *Book I*, by Bowers, Miller and Rourke.[27] Where specific reference to these texts is necessary in Table I, the initials MT will be used for *Mathematics for Today*, and MC for *Mathematics for Canadians*.

---

[25] *Op. cit.*, p. 31.

[26] M. E. LaZerte and William Betz, *Mathematics for Today* (Toronto: Ginn and Company, no date).

[27] Henry Bowers, Norman Miller and Robert E.K. Rourke, *Mathematics for Canadians*, *Book I* (Toronto: J. M. Dent and Sons (Canada) Limited, and The Macmillan Company of Canada Limited, 1947).

## TABLE I

## A COMPARISON OF THE GRADE IX MATHEMATICS TEXTBOOKS USED IN ALBERTA IN 1938 AND 1959

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Title | Mathematics for Today | Mathematics for Canadians, Book I |
| Authors | M.E.LaZerte, Professor of Educational Psychology and Director of the School of Education, University of Alberta | Henry Bowers, Principal of the Normal School, Stratford, Ontario |
| | | Norman Miller, Professor of Mathematics, Queens University, Kingston, Ontario |
| | William Betz | Robert E. K. Rourke, Headmaster and Instructor of Mathematics, Pickering College, Newmarket, Ontario |
| Publisher | Ginn and Company | Dent-Macmillan |
| Date of Publication | No date, but after 1934 | 1947 |
| Number of pages | 365 | 410 |
| Arrangement of Topics (Table of Contents) | Chapter | Chapter |
| | I. How Letters Are Used in Algebra | I. The Drawing of Accurate Figures |
| | II. The Equation | II. Length, Area and Volume |
| | III. Problems | III. The Beginning of Algebra |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Arrangement of Topics (Table of Contents, continued) | IV. Graphs | IV. The Equation and Its Uses |
| | V. Signed Numbers | V. The Formula and Its Uses |
| | VI. Fundamental Operations | VI. Integers |
| | VII. Equations of the First Degree in One Unknown | VII. Fractions and Decimals |
| | VIII-XIX. Geometry | VIII. Signed Numbers |
| | | IX. Graphs |
| | | X. More About Algebra |
| | | XI. Some Everyday Uses of Arithmetic |
| | | XII. In Which You Extend Your Knowledge of Geometry |
| Answers | None provided | Answers to all exercises are provided |
| Index | Provided | Provided |

BRIEF COMPARISON OF TREATMENT OF TOPICS

| | | |
|---|---|---|
| The Beginnings of Algebra | Chapter I<br>Geometrical illustrations are used to show how letters are employed in Algebra; e.g., an equilateral triangle of | Chapter III<br>Treatment grows out of formulas for areas and volumes in the preceding chapter |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| The Beginnings of Algebra (continued) | side s units has a perimeter of 3s units:<br>$P = 3s$ | |
| | Vocabulary is stressed | Vocabulary is stressed |
| | Formulas are introduced and used in the latter half of the chapter | |
| | Operations are explained using monomials only | Operations are explained using both monomials and binomials; e.g.,<br>$(2x + 5)(x + 3) = ?$<br>$(3a + 2b)^2 = ?$ |
| | Chapter summaries and end-of-chapter tests are provided | Review exercises are provided for each chapter |
| | | MC carries the early steps in Algebra farther than does MT in the first chapter devoted to it. |
| The Equation | Chapter II | Chapter IV |
| | Definition | Definition |
| | The axiom or assumption that both sides of an equation may be operated on in the same way is introduced and illustrated by a comparison with a labor- | The idea that both sides of an equation may be operated on in the same way is illustrated by drawing an analogy with a laboratory balance |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| | atory balance to prepare the student for methods of solution | |
| | Verbal type exercises in setting up simple equations are given | Exercises in "translating" verbal statements into symbolic statements and vice-versa are given |
| The Equation (continued) | This book gives more exercises at each step than does MC | The approach is somewhat more inductive than that of MT |
| | Equations involving fractions are introduced. The treatment is similar in both books, but MT has more exercises | Equations involving fractions are introduced |
| | The Rule of Transposition is given, with explanation | Transposition of terms is not mentioned |
| | Chapter III | Latter part of Chapt.IV |
| | Begins with specimen solutions | Begins with specimen solutions |
| | Steps in solution are outlined | Steps in solution are outlined |
| Problems | Exercises are classified as to type: practical, number relations, geometrical, business, miscellaneous | Exercises are arranged in order of difficulty: A, B, and C sets |
| | MT contains a type of mixture problem not encountered in the | Problems on per cent, interest and insurance are covered in |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Problems (continued) | present mathematics program until Grade XI (Mathematics 20) | the chapter entitled "Some Everyday Uses of Arithmetic" |
| Formulas and Their Use | The first algebraic concepts discussed in Chapter I are illustrated by exercises applied to formulas in the latter half of the chapter. Some of the formulas are derived ($A = s^2$, $V = e^3$), others are given ($V = \pi r^2 h$). Sample exercise: What is the area of a circle which has a radius of 3 in.? | Chapter II (52 pages) deals with Length, Area and Volume in the familiar plane and solid figures--the triangle, parallelogram, circle, rectangular solid, prism, cylinder and various combinations of these. The treatment is quite comprehensive<br>Chapter V. Exercises are given in making up formulas from tabulated data and from geometric figures with shaded portions<br>Changing the subject of a formula<br>Direct and inverse variation<br>The effect on the subject of a formula of changing certain of its variables in certain ways--a dynamic approach |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Integers, Fractions, Decimals | Nil | Chapters VI and VII give a comprehensive treatment<br>Integers. Hints on computing, and some short-cuts are given<br>Factors, multiples; prime and composite integers; powers and roots of integers are discussed<br>A good review of the meaning of fractions is given<br>Basic principles in operations with fractions are made clear; e.g., $an/bn = a/b$<br>Addition, subtraction, multiplication and division are reviewed<br>Reciprocals and complex fractions are explained<br>Decimal fractions are covered; also the operation of changing a common fraction to a decimal and vice-versa<br>Approximate numbers and significant figures are explained |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Signed Numbers | The idea of positive and negative numbers is introduced by various examples of increase and decrease, elevators, thermometers; the horizontal number scale | The treatment is similar to that of MT; the ideas are illustrated by elevators, thermometers, heights above and below sea level, deposits and withdrawals of money; etc. |
| | Addition and subtraction are made meaningful. The rules $(-a)(-b) = ab$ and $-a/-b = a/b$ are not given an adequate meaningful treatment | The idea of direction is stressed; the horizontal number scale is used to advantage. Operations with signed numbers are well explained. The rules $(-a)(-b) = ab$ and $-a/-b = a/b$ are given explanations which are not rigorous but which are clear and plausible |
| | The entire discussion is arithmetical and is confined to integers in this chapter | The rules are applied to literal numbers with illustrations and exercises |
| Fundamental Operations | Chapter VI<br>Signed literal numbers are introduced and practice is given in operating with them | No chapter is devoted to this per se; the treatment is distributed in other chapters in incidental ways |
| | Rules of exponents are given a quite formal treatment | Chapter X (More About Algebra)<br>Substitution is dealt with again |
| | The rules for removing parentheses are given in a relatively formal and detailed way | |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Fundamental Operations (continued) | This book stresses the multiplication of a polynomial by another polynomial more than does MC. An example:<br>$(2a^2 + 3 - 6a)(5a - a^2 - 4) = ?$ | Algebraic fractions are stressed |
| | Division of a quadrinomial by a binomial is included | Algebraic "long division" is not covered |
| Graphs | The topic is introduced by way of statistical graphs--pictographs, bar graphs, broken line graphs, pie charts | The appraoch is the same as in MT |
| | Graphs of formulas drawn from tables of values are adequately explained. Sufficient exercise in drawing linear graphs is provided | The approach here also is similar to that of MT |
| | Graphic solution of motion problems is covered | Non-linear types are included; e.g.,<br>$A = \pi r^2$;  $n = 36/t$ |
| Topics covered by one book only | Division of a quadrinomial by a binomial | A large amount of material on lengths, areas, volumes, with formulas is included. Emphasis is put on correct and consistent units in formulas |

TABLE I (continued)

| POINTS OF COMPARISON | THE 1938 TEXTBOOK (MT) | THE 1959 TEXTBOOK (MC) |
|---|---|---|
| Topics covered by one book only | | Rules are given for the number of digits to be retained in calculations with approximate numbers<br>Metric units of length and volume are given<br>Direct and inverse variation and the effect of changing the values of variables in formulas are taken up<br>Non-linear graphs are included; e.g., the graph of the function $\pi r^2$<br>A good deal of material on integers, fractions and decimals is provided by way of review and for the purpose of strengthening skills |

Commentary

With respect to the portions of the two textbooks which are relevant to this study (i.e., the sections on algebra), Table I shows that there is little difference in topical content. There are, however, differences in

the style of presentation and, perhaps more important, in the quantity of algebra to be found in the respective texts--which might result in a difference in the degree of expertness developed by the pupils who used them. Moreover, the two books seem to have been written to serve the needs of pupils of different academic attainments. The older textbook assumes a stronger arithmetic background in the student; it does not, like Mathematics for Canadians, include chapters on integers, common fractions and decimals, but presents the pupils immediately and uninterruptedly with algebra. Its style is more formal; it places more emphasis on manipulation. The newer textbook, on the other hand, provides a review and extension of arithmetical concepts before introducing the algebra, and alternates arithmetical with algebraic sections as it proceeds. It contains an abundance of expository material; the authors take great pains to explain principles carefully and thoroughly. Yet it may be that this is largely a waste of effort except in so far as it helps the teacher, for students at the junior high school level do not read their textbooks extensively. At the same time the number of exercises is sometimes too small to ensure mastery of the techniques. An illustration might be given. In the final chapter devoted to algebra, the addition and subtraction of algebraic fractions is introduced. Few man-

ipulations are more troublesome to students, yet the text provides only nine exercises for immediate practice, and only two more in the review set. In the same chapter, equations involving signed numbers are brought in for the first time and are complicated by the use of fractional coefficients and constants; yet there are only ten practice exercises, with five others in the review set.

In contrast, Mathematics for Today condenses the explanation and requires the student to plunge quickly into the exercises. There is something to be said in favor of this. The best source of explanation is a good teacher, whose elucidations and illustrations are adapted to the background and needs of his particular class. And since principles become meaningful and skills automatic through application and practice, the learner should begin the practice without undue delay and continue it to the point of mastery.

A count revealed that in the sections of Mathematics for Canadians which deal with algebra there are approximately 815 exercises for written answers in 160 pages of text, an average of just over 5 exercises per page. Mathematics for Today has approximately 1625 exercises in the 216 pages of Part I (Algebra for Today), an average of 7.5 exercises per page.

The general impression received from a consideration of the two textbooks as vehicles for teaching the Grade IX mathematics course is that the teacher would find it easier in practice to make up the deficiencies of the older book than to make up those of the newer. The teacher can supply additional explanation and illustration (that is his function) more readily than he can supply additional exercises (which it should not be necessary to do). Mathematics for Canadians tends to spread itself too thinly. Topically it is closely comparable to the older book, but quantitatively its algebra content is smaller. Its lower ratio of exercises to explanation and its considerably smaller total number of exercises suggest that the students who use this book receive less practice than did those who used Mathematics for Today. This might lead to the expectation of a lower performance standard in 1959 than that which existed in 1938.

# CHAPTER IV

# INITIAL PROCEDURE, THE TESTS, AND THE SAMPLES

## I. INITIAL PROCEDURE

The first step in getting the investigation under way was to obtain permission from the Department of Education and the Edmonton Public School Board to make use of their sources of data. This permission was granted generously. The Examinations Branch of the Department had no objection to the reproducing and use of the required tests or to the researcher's unrestricted access to any of their records which might prove useful. The Director of Personnel and Research of the Edmonton public school system explained that any testing of pupils for research studies was governed by certain conditions of policy respecting the usefulness of the project to the system, the extent to which it might interfere with school routine, etc. The project qualified under the given conditions, and permission was granted. It remained then to enlist the co-operation of the principals and teachers in the junior high schools which had been selected for the sample (the sampling method is described in the latter part of this chapter). Outlines of the

proposal and requests for co-operation were mailed to the principals.[28] With one exception the replies indicated a keen interest and a willingness to participate. A comparable substitute was readily obtained for the school which was unable to take part. After these arrangements had been completed, about 600 copies of the tests were prepared, packaged, and delivered to the schools, together with instructions for supervision during the writing period.[29] The principals were asked to give the tests as late in June as possible, and not before the 11th or 12th of the month. When the tests had all been administered, the answer papers were picked up at the schools.

## II. THE TESTS

It has been the practice of the Alberta Department of Education for many years to include a test of general ability (formerly called a General Test) in the set of final examinations for Grade IX. The writer felt that the use of the 1938 General Test in conjunction with the test in algebra would give a good comparison of the

---

[28] See Appendix B, p. 126.

[29] See Appendix C, p. 129.

abilities of the students in the two samples--possibly a better comparison than I.Q. scores, though the latter were to be used as well. Hence the students were required to write the General Test in addition to the test in algebra.

## The Algebra Test[30]

The 1938 Departmental Examination in Algebra was a one-and-a-half-hour test covering the major topics of the course. It was printed on a single sheet, and foolscap was provided for the students' answers. Specific instructions were given to the effect that full solutions were to be shown for certain questions. Checks were also required for some of these.

In setting up the test for the 1959 students, three alterations were made. The first was the prefixture of a short message to the students, explaining the purpose of the test. The second was a change in format: the questions were spaced out on five pages, eight and one-half by fourteen inches in size, to provide room for solutions and rough work and thus to eliminate the need for foolscap. The third change was the re-locating of the question

---

[30] See Appendix D, p. 132.

originally numbered 14; it was moved to the end of the paper (Number 20). This change was made because the question required an operation not included in the 1959 course--the division of a quadrinomial by a binomial. The writer decided to retain the question, but felt that if it were left in its original place some students might waste valuable time puzzling over it; putting it last ensured that they would not encounter it until they had completed the rest of the paper.

Except for the question to which reference has just been made, the content of the paper was in close agreement with the 1959 course. Question 11 involved successive multiplications resulting in the removal of two pairs of parentheses, one within the other. This particular type is not specifically dealt with in the 1959 course, but many students performed the operation successfully by applying basic principles. Question 19 required the reading of a type of graph with which the 1959 students were probably less familiar than were the 1938 students; but it was straightforward, and in fact turned out to be one of the better-answered questions.

### Analysis of the Algebra Test

The question of *what this Algebra test actually*

measures has a strong bearing on the conclusions of this study. To be consistent with the aims given in the 1938 Programme of Studies it should measure primarily the ability of the pupils to interpret the environment rather than to perform computations and manipulations. A consideration of the test's content indicates, however, that it is a measuring instrument mainly of manipulative skills and of recall. Questions 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14 and 16 are entirely of the manipulative type. Questions 8 and 9 require analysis and manipulation but not interpretation. Question 10 requires the ability to make a partial translation from words to symbols; this is chiefly a language problem and is of a type for which practice was provided in the course. Question 15 asks the pupil to express the volume of a cylinder of given base in terms of its height, first by a formula and then by a graph. This is a straight recall type. The wording of the question suggests the proper way to label the axes, and the extent of the scale for the independent variable is given. This could hardly be called an interpretative question; it merely asks the pupil to do something which he had previously learned and practised. Question 17 is once again a language exercise: it requires a re-statement of a word problem as a conditional equation. Number 18 is a problem which requires analysis, a translation

from verbal statements to an equation, and manipulation and computation to solve the equation and get the answer. It is a typical textbook problem, involving no original interpretation. Question 19, which shows graphically the journeys of two cars concurrently travelling the same road in opposite directions, does require interpretative ability of the graph-reading kind; but the graph is of a type familiar to the student through his textbook exercises. Question 20 is purely manipulative.

This brief analysis is summarized in Table II, page 50, which shows the percentage weighting of the paper in the learnings classified in Column (1). It points to the conclusion that the Algebra test was an instrument for measuring (1) manipulative and computational skills, (2) the ability to commute words and symbols, (3) the ability to analyse a problem and set up a solution, (4) recall of techniques, (5) the ability to interpret a familiar type of graph. The greatest weight was placed on the manipulative skills: considering the paper as a whole, the routine techniques of arithmetic and algebra (including graph-drawing) accounted for the major portion of the marks.

## TABLE II

## ANALYSIS OF THE 1938 GRADE IX DEPARTMENTAL EXAMINATION IN ALGEBRA, ON THE BASIS OF THE KINDS OF ABILITIES TESTED

| Learning ability or skill tested (1) | Question numbers (2) | Total value (3) | Per cent of total score for each kind of learning in Column (1) (4) |
|---|---|---|---|
| Purely manipulative | 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 16, 20 | 33 | 44.0 |
| Language skills | 10, 17 | 11 | 14.7 |
| Combination of analytic, language, manipulative and computational skills | 8, 9, 18 | 11 | 14.7 |
| Graph work (involving recall, analysis and manipulation | 15 | 7 | 9.3 |
| Ability to interpret (a graph) | 19 | 13 | 17.3 |
| Totals | | 75 | 100.0 |

### The General Test

This was a forty-five minute general ability test requiring the candidate to use both verbal and computational skills in making generalizations, drawing inferences, detecting analogies, performing calculations and solving problems.[31] It was entirely objective. At the beginning were three typical questions together with their answers, illustrating the way in which the paper was to be done. The test was set up for the 1959 students exactly as it was in 1938, except that it was mimeographed on eight and one-half inch by fourteen inch paper and, like the Algebra test, carried a message to the student on the first page.

## III. THE SAMPLES

### The 1938 Sample

The 1938 sample was a random sample of 203 individuals (represented by their record cards) drawn from a parent population of 1292 students comprising thirty-five classrooms in fifteen schools. The names of the entire 1292 students were copied from a list in the Department of Education files, in the order in which they occurred there. This list was the original compilation

---

[31] See Appendix F, p. 144.

received by the Department from the city schools, with the names arranged alphabetically by classrooms. The names were numbered consecutively as they were copied. The sample was then drawn by means of a table of random numbers. It represented approximately 16 per cent of the population.

The names which had been thus selected were transferred to a form having columns for recording sex, age, birthdate, I.Q., raw score in Algebra, raw score in the General Test, year's attendance, grade repeater or non-repeater, final mathematics grading and pass category. These data were all obtained from the record cards in the Department's files.

### The 1959 Sample

Ideally, the 1959 sample should also have been random, drawn in the same way as that of 1938. However, this was not possible. The selecting of this sample was dictated by considerations of practicability rather than by the requirements of scientific rigor. Such is generally the case in studies of this kind, but it does not follow *ipso facto* that the results are invalid. In selecting the sample, the investigator had a choice of : (1) having every Grade IX student in the system write

the tests, and then drawing a random sample from the answer papers; (2) taking a random sample of the eighty-eight classrooms; (3) taking a random sample of the twenty-eight schools; (4) drawing a purposive sample of schools with a view to having it as representative as possible. The first possibility was discarded because of the magnitude and expense of the undertaking, and because such a massive testing program was unnecessary. The second plan was ruled out as inadvisable: to ask that single classrooms all over the city be isolated for testing purposes would have raised an unreasonable administrative problem for the principals, chiefly because of the universal departmentalization of instruction in the junior high schools. The third method was rejected on the ground that a small random sample stands a strong chance of being unrepresentative. As Parten points out in Surveys, Polls and Samples, "The possibility of obtaining a poor or misleading sample is always present when random selection is used."[32] This is especially true if the sample is small. "If the sample is not large enough, there is a chance that it may be an extreme deviate, and therefore not representative."[33]

---

[32] Mildred Parten, Surveys, Polls and Samples (New York: Harper and Brothers, 1950), p. 225.

[33] Ibid., p. 220.

"As the random sample is enlarged, it becomes increasingly representative of the universe."[34]

In view of these considerations it was decided to adopt course (4) and make a purposive selection of schools; all Grade IX students in each school would then write the tests together, with a minimum of interference with daily routine. Both practicability and logic therefore prescribed the sampling method. Further justification of the course taken is supplied by Parten:

> . . . the sampling procedure should be directed more toward finding and using practical devices to avoid distorted samples than toward using refined mathematical formulas to measure chance fluctuations in sampling.[35]
>
> . . . if the choice must be made between employing a logical, non-mathematical approach which might do no more than guarantee that the sample is unbiased, and using a theoretically efficient sampling design which in practice might permit the introduction of biased errors, the author believes the former is preferable. It is better to have a large sample of the right thing than just the right size of the wrong.[36]
>
> Purposive selection of communities in nation wide studies is often necessary because of administrative considerations.[37]

---

[34] Ibid., p. 225.

[35] Ibid., p. 113.

[36] Ibid., p. 113-4.

[37] Ibid., p. 239.

In selecting the sample of schools, four criteria were used, the first two being closely associated: (1) the schools were to be uniformly distributed geographically; (2) a balance was to be sought between schools located in the older parts of the city and in the more recently settled residential areas; (3) each school should have approximately the average number of classrooms for the junior high schools of the city; (4) the size of the sample should be between 400 and 500 students--roughly from 15 per cent to 20 per cent of the population. The first two of these conditions were to ensure representativeness in socio-economic level; the third to obtain a typical degree of departmentalization of instruction; and the fourth to secure a sample of relatively the same size as that of 1938.

It was recognized at the outset that a school which was chosen might fail to co-operate, and this did in fact happen in one instance, regrets being expressed by the principal. Using these four guiding considerations, then, and making one substitute choice, the following schools were selected:

KING EDWARD PARK, a modern type school (built in 1949) with four Grade IX rooms, in a district settled

mainly since 1945;

KING EDWARD, an old school (built in 1913) with three Grade IX rooms, in a semi-industrial area;

McKERNAN, a fairly new school (1951) with four Grade IX rooms, in a recently-settled district;

McKAY AVENUE, probably the oldest school in Edmonton (1904), having one Grade IX room, located on the edge of the main business district, in almost the exact centre of the city;

McDOUGALL, an old school (1913) with three Grade IX rooms, having a long-established semi-industrial and business area on the south and east and a more recently-developed residential area to the north and west;

WESTMOUNT, an old school (1913) with three Grade IX rooms, in a district similar to that of McDougall.

These six schools housed a total of eighteen Grade IX classrooms, accommodating about 550 students. Their locations are shown on the map, Figure 1, page 57.

Representativeness of the sample. Since the 1959 sample was not random, the question of its representativeness was present from the beginning. It was drawn from districts which collectively were representative of the residential areas of the city, and the constituent schools

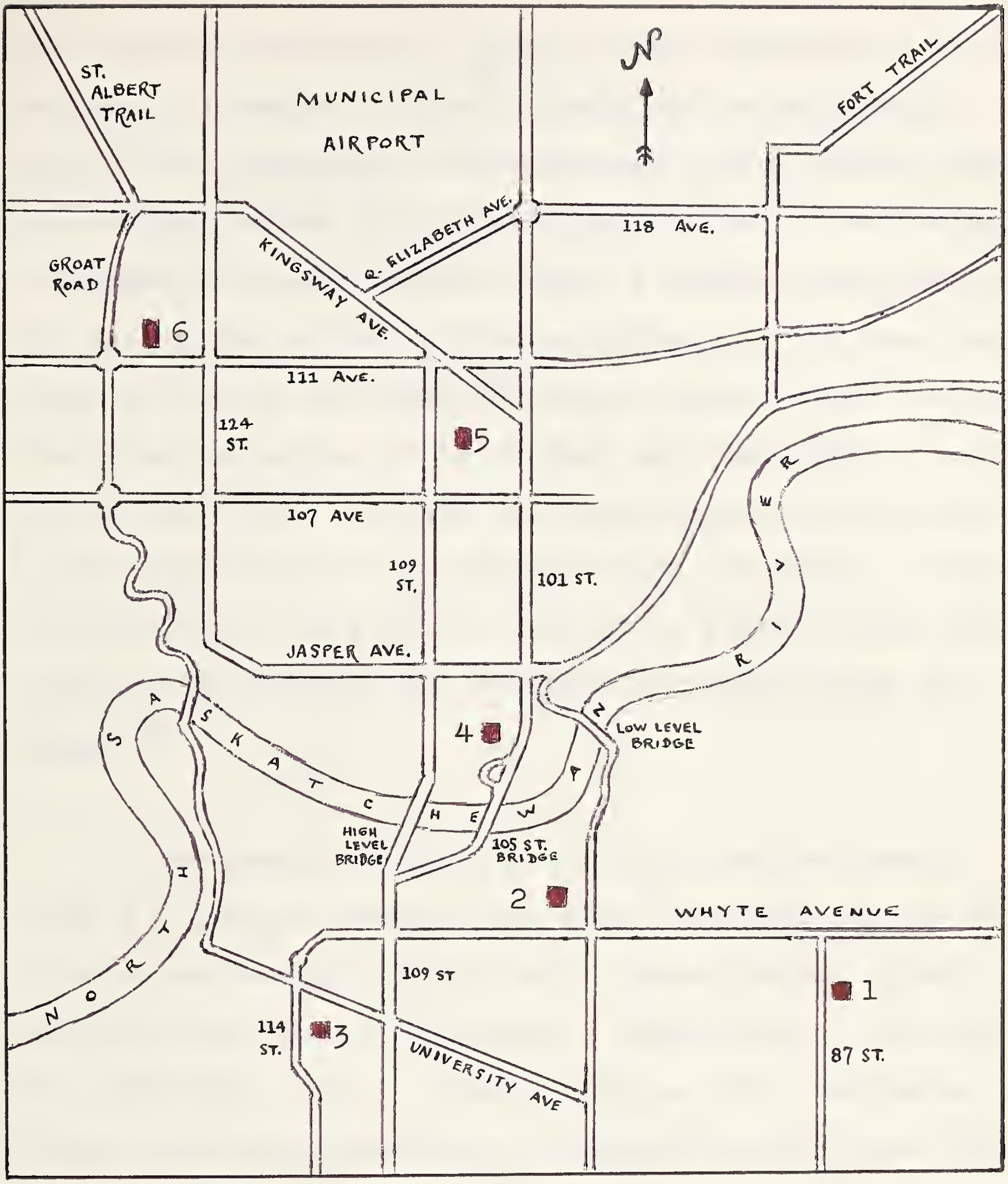


FIGURE 1. SKETCH MAP OF THE CITY OF EDMONTON SHOWING THE LOCATIONS OF THE SIX SCHOOLS USED IN THIS STUDY

| | |
|---|---|
| 1. King Edward Park | 4. McKay Avenue |
| 2. King Edward | 5. McDougall |
| 3. McKernan | 6. Westmount |

as a group exhibited the instructional features of the system as a whole. It would therefore be expected to typify the population. Nevertheless a more precise measure of the degree of similarity of the sample to its parent population was desired. Such a measure was furnished by the scores on the 1959 Departmental Ability Test, which were on file in the Edmonton Public School Board offices. The separate scores on the Verbal and Quantitative sections of the test were combined for each student and distributions drawn up for the population and the sample. These distributions are given in Table III, page 59, with such products and sums as are required for calculating the means.[38]

The population mean $\mu = \sum xf_p/M$ and the sample mean $\bar{x} = \sum xf_s/N$, where M and N are the sizes of the population and sample respectively. Substituting appropriately from Table III gives $\mu = 188411/2512 = 75.0$, and $\bar{x} = 38230/505 = 75.7$. Ninety-nine per cent confidence limits were calculated for the population mean from the

---

[38] Discrepancies between the total number of pupils in both the sample and the population shown in Table III and figures given elsewhere in the study are the result of absences at the time of the Algebra test and the General Test and of absences, transfers and drop-outs toward the end of the school year.

## TABLE III

## DISTRIBUTION OF SCORES ON THE GRADE IX ABILITY TEST FOR THE 1959 POPULATION AND SAMPLE

| Class Interval | Class Mark $x$ | Frequency (population) $f_p$ | Frequency (sample) $f_s$ | $xf_p$ | $xf_s$ |
|---|---|---|---|---|---|
| 21-25 | 23 | 1 | 1 | 23 | 23 |
| 26-30 | 28 | 7 | 1 | 196 | 28 |
| 31-35 | 33 | 12 | 5 | 396 | 165 |
| 36-40 | 38 | 43 | 10 | 1634 | 380 |
| 41-45 | 43 | 55 | 9 | 2365 | 387 |
| 46-50 | 48 | 88 | 20 | 4224 | 960 |
| 51-55 | 53 | 135 | 19 | 7150 | 1007 |
| 56-60 | 58 | 179 | 42 | 10382 | 2436 |
| 61-65 | 63 | 190 | 33 | 11970 | 2079 |
| 66-70 | 68 | 224 | 46 | 15232 | 3128 |
| 71-75 | 73 | 286 | 51 | 20878 | 3723 |
| 76-80 | 78 | 249 | 41 | 19422 | 3198 |
| 81-85 | 83 | 278 | 62 | 22874 | 5056 |
| 86-90 | 88 | 285 | 46 | 25080 | 4048 |
| 91-95 | 93 | 212 | 47 | 19716 | 4371 |
| 96-100 | 98 | 163 | 38 | 15974 | 3724 |
| 101-105 | 103 | 89 | 31 | 9167 | 3193 |
| 106-110[β] | 108 | 16 | 3 | 1728 | 324 |
| | | 2512 | 505 | 188411 | 38230 |

---

[β] The possible score was 110.

sample statistics and were found to be 73.7 and 77.7. The true population mean (75.0) lies between these limits; consequently there is no significant difference between the population and sample means.[39]

The standard deviations of the population and the sample were calculated also and were found to be $\sigma = 16.52$, $s = 17.45$. A test indicated that the difference between the standard deviations is not significant (the calculations may be found in Appendix H, p. 173).

These tests having been carried out, it may be said that since the sample mean and standard deviation are the same (with a satisfactory degree of probability) as those of the population, the sample is closely representative.

Visual evidence that the distributions of the scores for the population and the sample are the same is given by the frequency polygons, Figures 2 and 3, pages 61 and 62. It will be noted that the two polygons have the same configuration, the "indentations" of the sample polygon being amplified by the coarser vertical scale.

---

[39] The statistical methods used here are described in Chapter VI, and the detailed calculations are given in Appendix H, p. 168 ff.

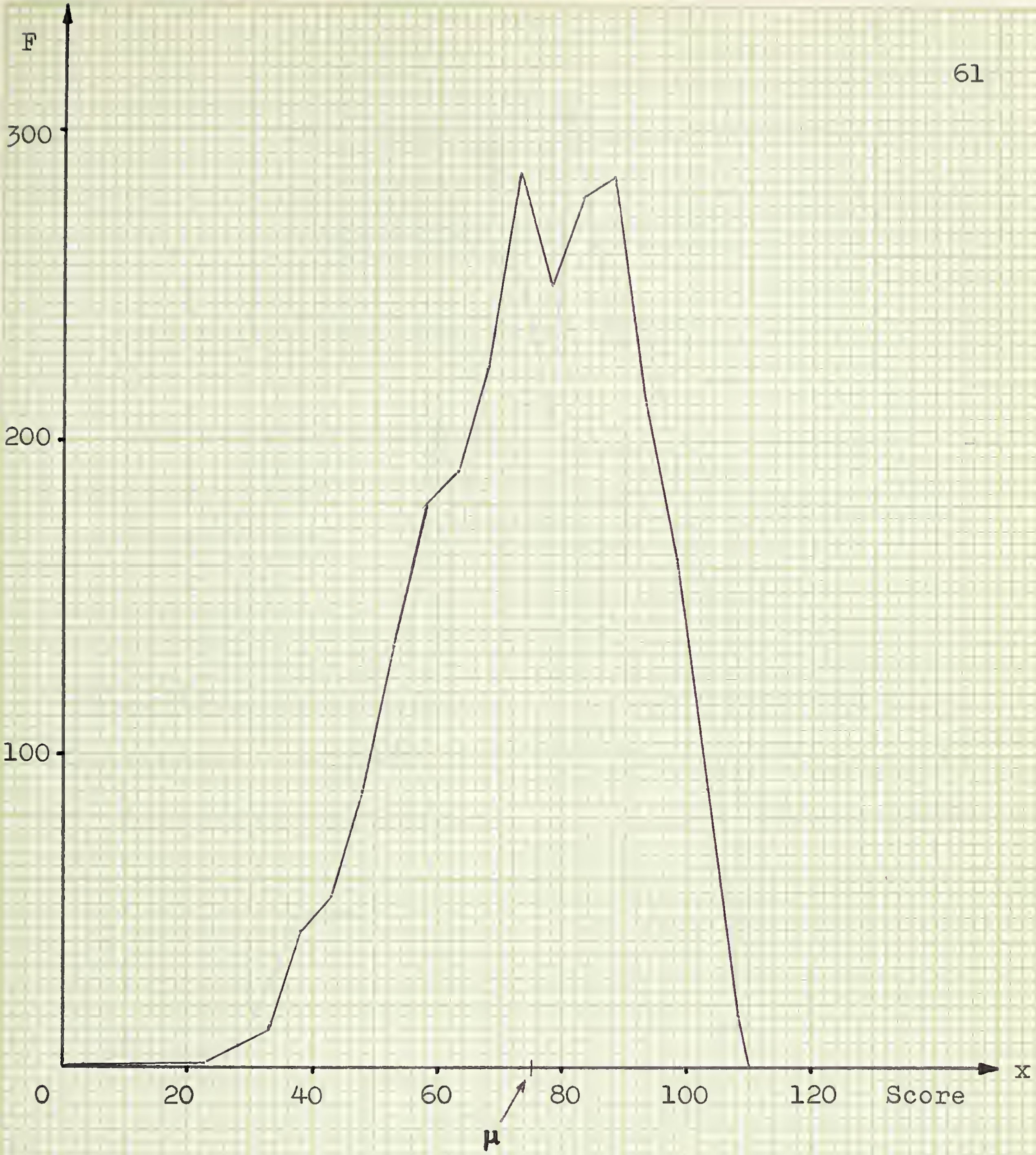
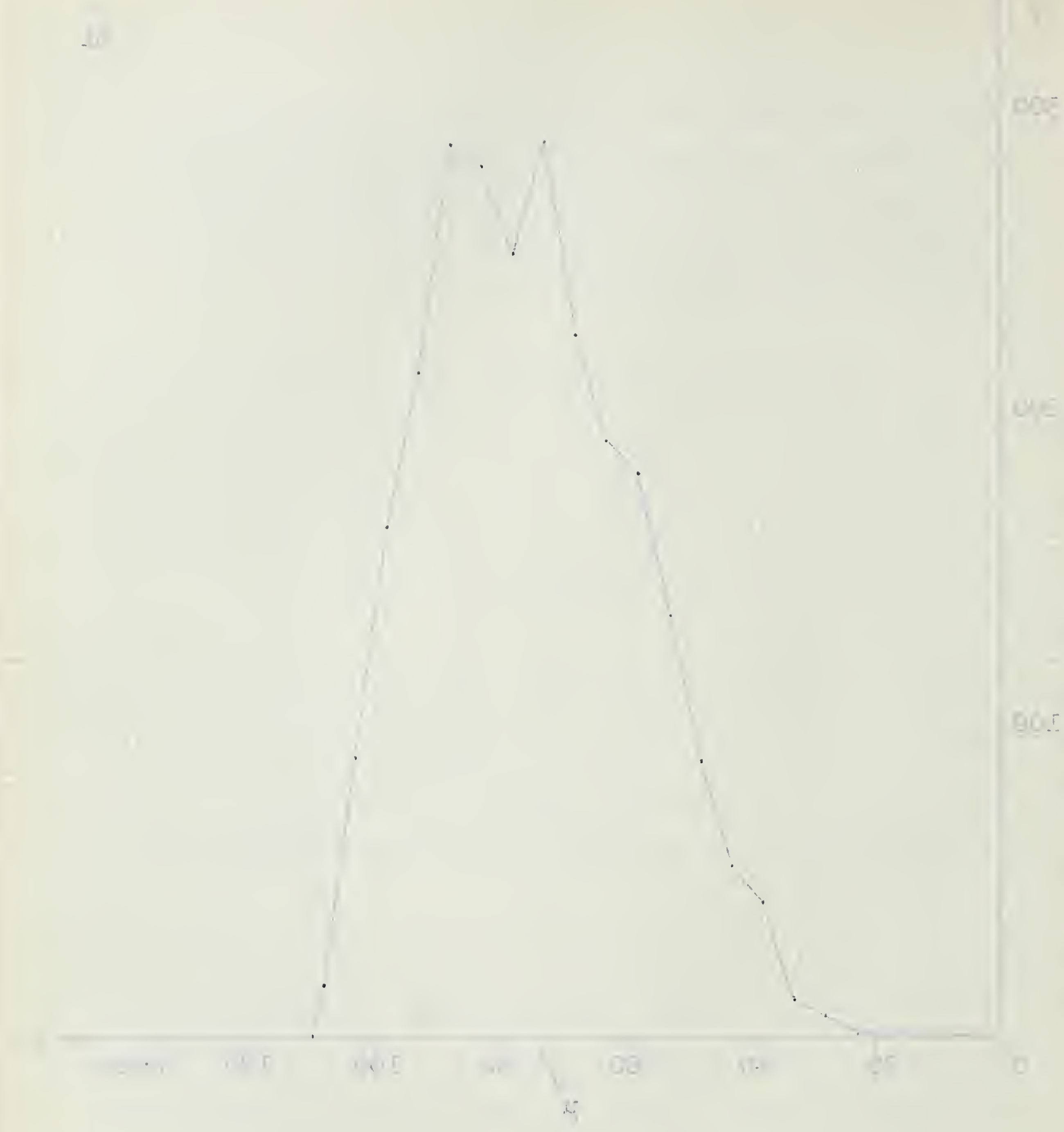


FIGURE 2. FREQUENCY POLYGON SHOWING THE DISTRIBUTION OF SCORES ON THE GRADE IX ABILITY TEST FOR THE 1959 POPULATION



[illegible]

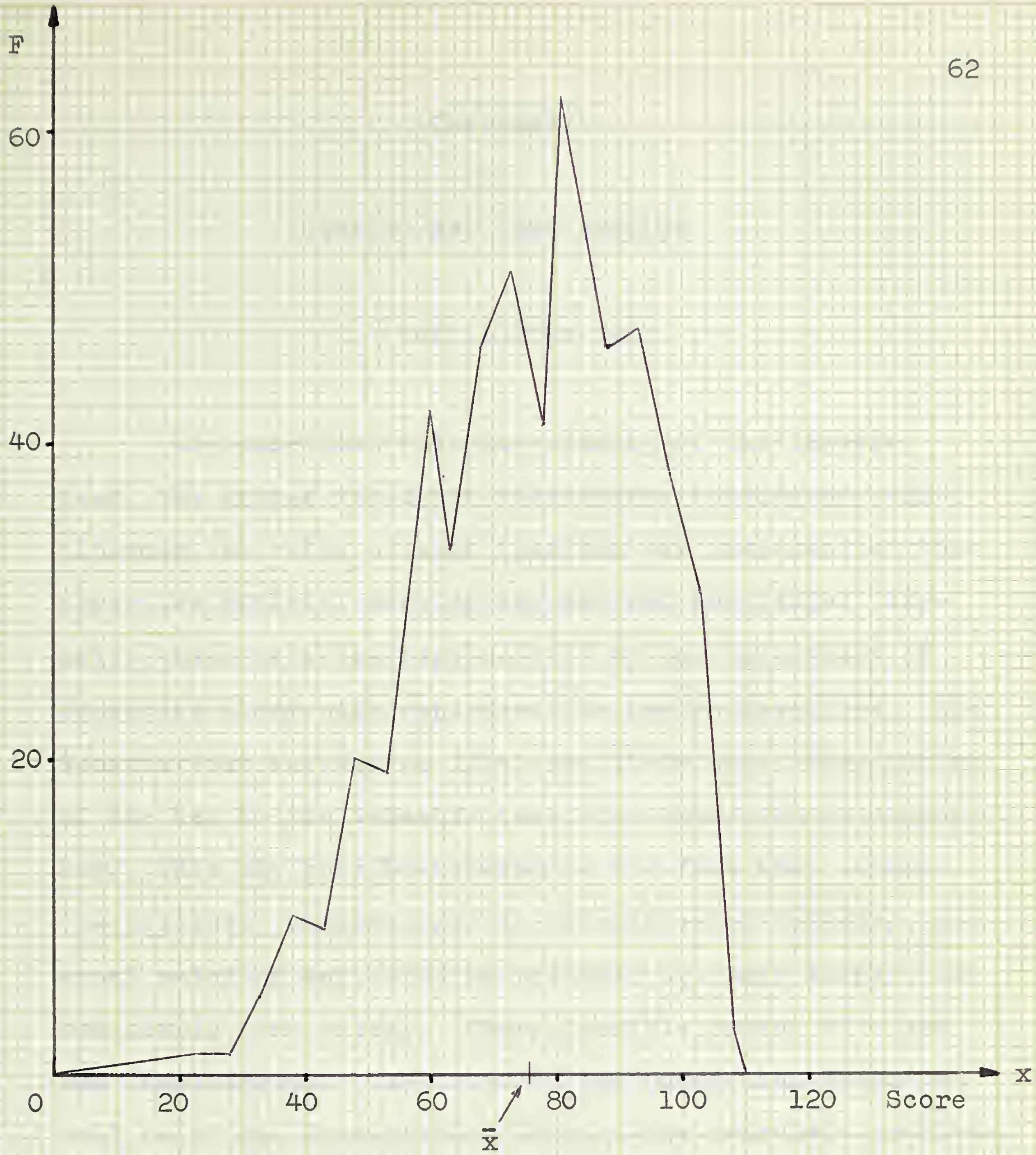


FIGURE 3. FREQUENCY POLYGON SHOWING THE DISTRIBUTION OF SCORES ON THE GRADE IX ABILITY TEST FOR THE 1959 SAMPLE

# CHAPTER V

# SCORING AND TEST RESULTS

## I. THE ALGEBRA TEST

In connection with the scoring of the Algebra test, the writer faced the unfortunate contingency that, although the value of each question was indicated on the paper, an official scoring key was not available. Normally these keys are kept on file by the Department of Education along with copies of the tests themselves. The General Test key was on file, and there were three copies of the key to the Geometry test; but there was no Algebra key. This may have been owing to the fact that, since the students' answers were to be written on foolscap, no exact matching key could be printed; or, more likely, it had simply gone astray. Every possible source of a key was investigated. The files of the Curriculum Branch as well as of the Examinations Branch were searched; archives in the sub-basement store-rooms were examined; dead files in different government buildings were consulted. The former Supervisor of Examinations, now retired, was asked for help, as were former filing clerks. The Department payrolls for 1938 were looked up in the hope of finding

there a separate list of sub-examiners, one of whom might still have a key. Appeals were made to teachers who had taught Grade IX classrooms in 1938. Teachers in other parts of the province were contacted by mail in the hope that someone might be able to exhume a key. None was found.

It is probable that even if there had been an official key in the Department's files it would not have been perfectly reliable. The writer is aware from his own experience as a sub-examiner of Departmental examinations that the scoring keys as provided are sometimes modified by the marking committees in the interests of satisfactory evaluation. The committees have this right, subject to the approval of the supervising examiner. The usual practice of marking committees is to spend the first half day or more discussing, criticizing and if necessary altering the keys. Even after this preparation there are occasions when a student's answer requires special consideration and a scoring decision independent of the key's prescription. Thus it is submitted that the writer could not have relied with complete confidence on a printed key even if one had been available, unless it had been a copy actually used by an examiner, bearing his changes and notations.

At any rate the investigator had no alternative but to construct his own key.[40] Some of the test questions had a value of only one mark; for others the manner of distributing the two or three possible marks was obvious. It was in a few cases only that it was necessary to make an independent assignment of marks for the various steps in the problem's solution. This was done with an aim to distributing the marks with appropriate relative weight among the essential steps in the solution. The writer's judgment in making these apportionments was based on the experience of several terms of service as a subexaminer of Departmemtal examinations in mathematics. He submits that any discrepancy between the key used by the examiners in 1938 and the one used to score the papers in 1959 would be very slight.

## II. THE GENERAL TEST

The General Test key was on file at the Examinations Branch of the Department of Education. There were no uncertainties here with respect to question values: each correct answer was worth one mark.

---

[40] See Appendix E, p. 139.

## III. SUMMARY OF RESULTS

Table IV gives the mean raw scores for the two samples on the Algebra test and on the General Test. The most notable fact revealed by these figures is that the 1959 students scored higher on the General Test but lower on the Algebra than did the students of 1938. The possible explanations of this are discussed in Chapter VII.

TABLE IV

MEAN RAW SCORES AND PERCENTAGES ON THE ALGEBRA TEST AND THE GENERAL TEST FOR THE 1959 AND 1938 SAMPLES

| | ALGEBRA TEST | | GENERAL TEST | |
|---|---|---|---|---|
| | RAW SCORE | % | RAW SCORE | % |
| Possible score | 75 | 100 | 89 | 100 |
| 1959 Sample | 26.66 | 36 | 60.78 | 68 |
| 1938 Sample | 31.36 | 42 | 56.49 | 63 |

A comprehensive picture of the relative standings of the two samples in general ability and in algebraic proficiency, and of the standing of the 1959 sample in each of these areas is given by the frequency polygons, Figures 4, 5 and 6, pages 68, 69 and 70. Figure 4 shows the superiority of the 1959 sample in general ability; Figure 5 shows the inferiority of this sample in algebraic

achievement. Figure 6 shows the wide discrepancy between the general ability of the 1959 sample and its level of attainment in algebra.

After the Algebra papers had been scored, a count was made of the number of marks obtained by each student on each question. This count is shown in Table V. The table could be used in conjunction with the test paper to make a diagnosis of pupil difficulties. Although such an analysis is not an aim of the present study, two or three remarks may be made. The large number of students who scored zero on the majority of the questions indicates a general weakness in algebra. This has already been pointed up by the mean score of 36 per cent for the group as a whole. With respect to the relative difficulty of the questions, it is clear that Question 9 was the most troublesome:

> "The price of a bicycle is increased x%. If the new price is $40, what was the old price?"

The scores on Question 20 deserve some comment. This is the question which was moved to the end of the paper because it involves algebraic "long division", an operation not included in the 1959 course:

> "What is the remainder when $20a^3 - 22a^2 + 33a - 18$ is divided by $5a - 3$?"

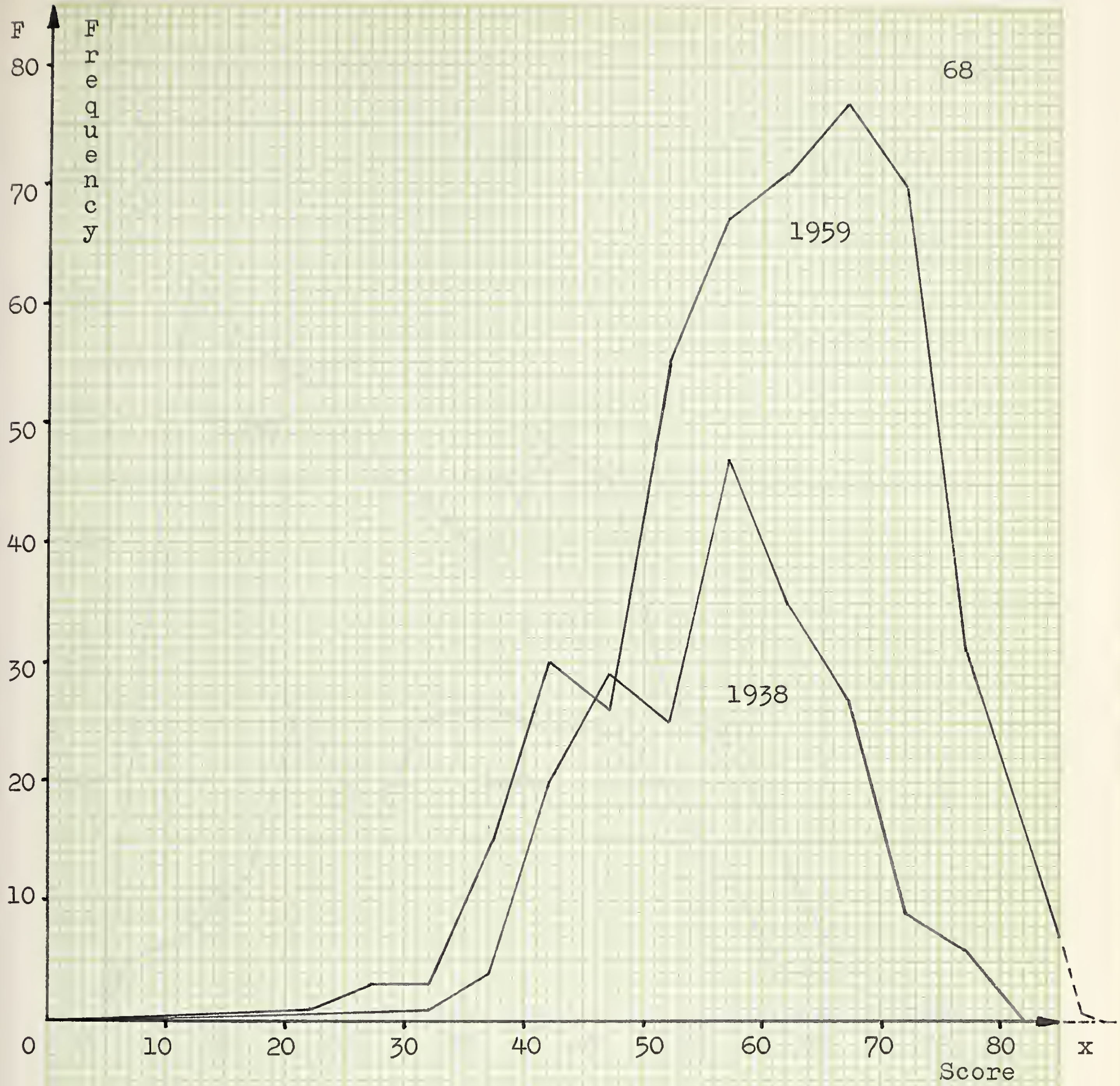


FIGURE 4. FREQUENCY POLYGONS SHOWING THE DISTRIBUTION OF SCORES ON THE GENERAL TEST FOR THE 1959 AND 1938 SAMPLES

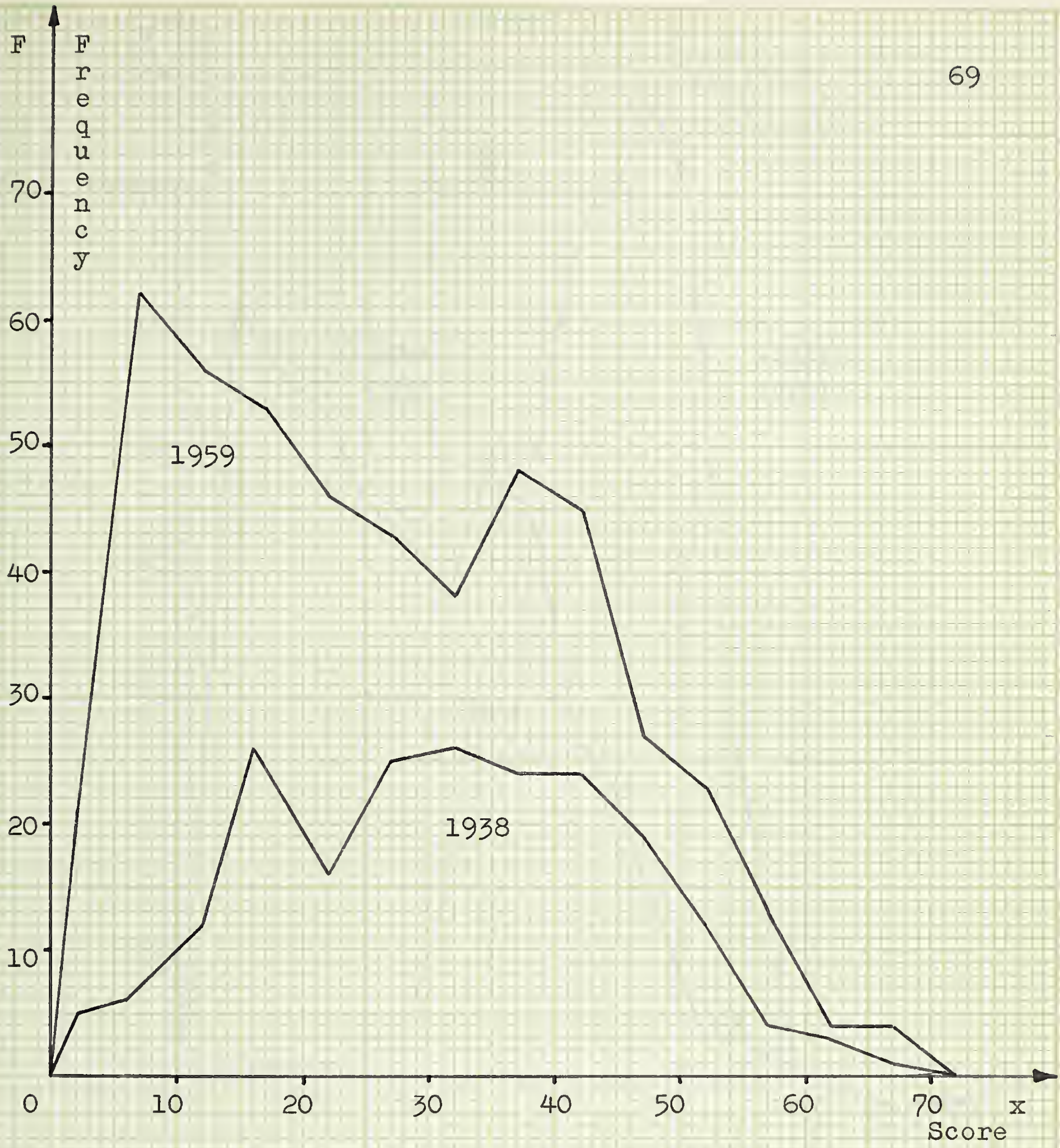


FIGURE 5. FREQUENCY POLYGONS SHOWING THE DISTRIBUTION OF SCORES ON THE ALGEBRA TEST FOR THE 1959 AND 1938 SAMPLES

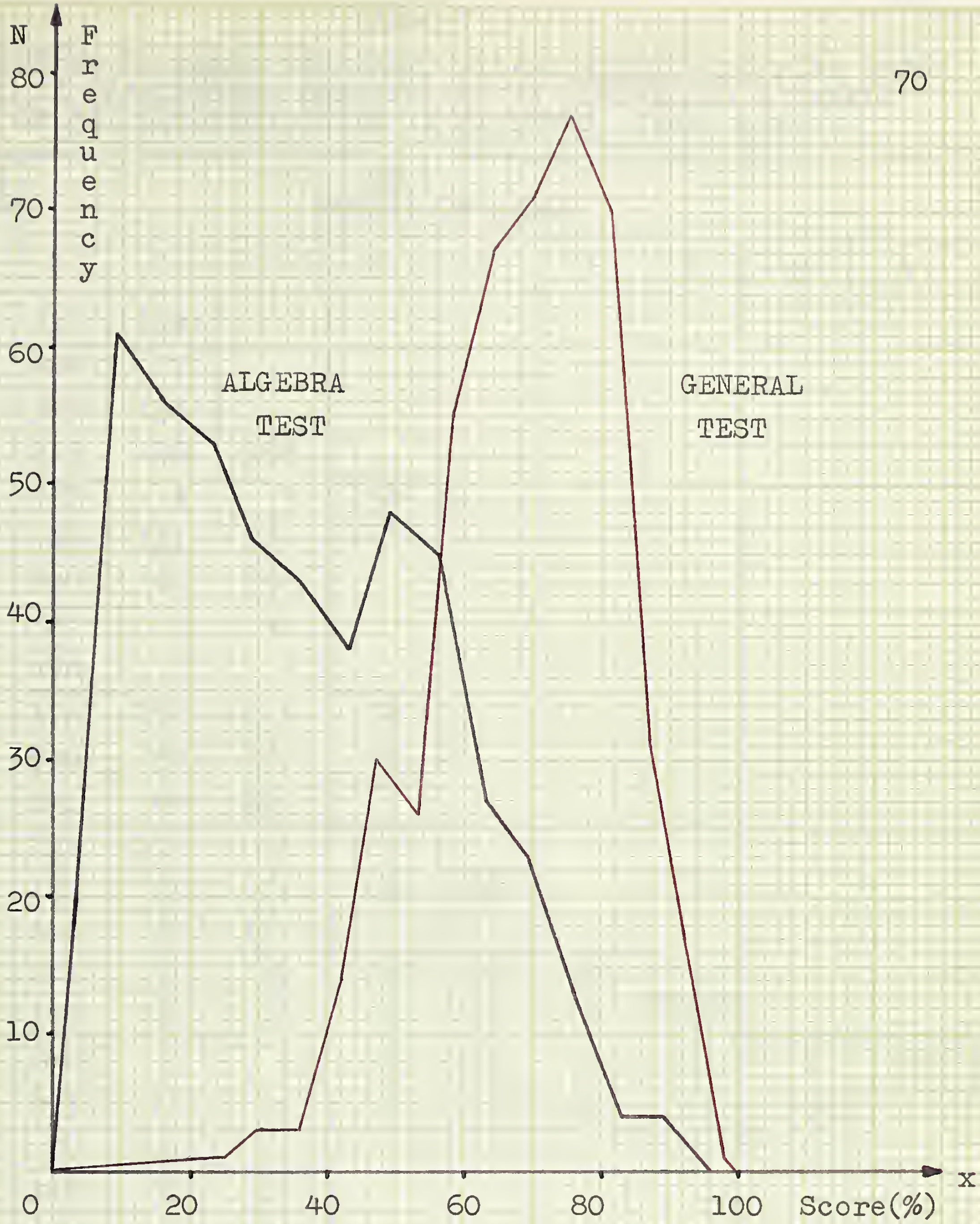


FIGURE 6. FREQUENCY POLYGONS SHOWING THE DISTRIBUTION OF SCORES ON THE GENERAL TEST AND THE ALGEBRA TEST FOR THE 1959 SAMPLE

[illegible]

[illegible]

[illegible]

## TABLE V

## THE NUMBER OF MARKS OBTAINED ON EACH QUESTION OF THE 1938 GRADE IX DPARTMENTAL EXAMINATION IN ALGEBRA BY A SAMPLE OF 481 EDMONTON GRADE IX STUDENTS, JUNE, 1959

| QUESTION | NUMBER OF MARKS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 317 | 164 | | | | | | | |
| 2 | 424 | 57 | | | | | | | |
| 3 | 343 | 138 | | | | | | | |
| 4 | 213 | 86 | 182 | | | | | | |
| 5 | 337 | 3 | 7 | 134 | | | | | |
| 6 | 84 | 121 | 276 | | | | | | |
| 7 | 207 | 44 | 230 | | | | | | |
| 8 | 387 | 0 | 94 | | | | | | |
| 9 | 478 | 0 | 3 | | | | | | |
| 10 | 176 | 153 | 113 | 39 | | | | | |
| 11 | 320 | 52 | 27 | 82 | | | | | |
| 12 | 201 | 64 | 216 | | | | | | |
| 13 | 205 | 107 | 45 | 124 | | | | | |
| 14 | 122 | 23 | 33 | 62 | 241 | | | | |
| 15(a) | 344 | 20 | 117 | | | | | | |
| 15(b) | 271 | 51 | 30 | 14 | 30 | 85 | | | |
| 16 | 356 | 36 | 10 | 4 | 12 | 1 | 62 | | |
| 17 | 240 | 0 | 1 | 0 | 146 | 0 | 0 | 0 | 94 |
| 18 | 293 | 16 | 8 | 7 | 12 | 45 | 31 | 69 | |
| 19(a) | 50 | 431 | | | | | | | |
| 19(b) | 345 | 50 | 86 | | | | | | |
| 19(c) | 367 | 7 | 107 | | | | | | |
| 19(d) | 251 | 55 | 175 | | | | | | |
| 19(e) | 30 | 24 | 144 | 283 | | | | | |
| 19(f) | 181 | 48 | 27 | 225 | | | | | |
| 20 | 466 | 1 | 1 | 13 | | | | | |

Note: The heavy line indicates the possible score for each question.

The table shows that thirteen students performed the division correctly. Nine of these were from one school and four from another, which suggests that this type of division may have been taught to these students incidentally or as an enrichment topic.

# CHAPTER VI

# STATISTICAL ANALYSIS OF DATA; TESTS OF SIGNIFICANCE

## I. STATISTICAL THEORY

The crucial question of this study is whether the differences in the mean raw scores of the 1959 and 1938 samples on the Algebra test and the General Test are statistically significant. The tests of significance which are used in this chapter are described in a standard textbook, _Mathematics of Statistics_, by Kenney and Keeping.[41] A brief summary of the pertinent sections is given below. Further details may be found in Appendix G, p. 152.

_Test 1. Confidence limits for the population mean._ One of the principal uses of the mathematics of statistics is to enable researchers to draw reasonably valid inferences about certain characteristics of a population from information obtained from a sample. The statistics chiefly used in the present study are the mean raw scores of two samples of students on two written tests.

---

[41] J. F. Kenney and E. S. Keeping, _Mathematics of Statistics, Part One_ (Toronto: D. Van Nostrand Company (Canada) Ltd., 1954).

It is important to determine the probability that the mean raw scores of the respective parent populations would be the same as those of the samples. The determination of this probability is the purpose of the first statistical test.

It was shown by W. S. Gosset, writing under the pen-name of "Student", that the variable

$$t = \frac{\bar{x} - \mu}{s/(N - 1)^{1/2}} \qquad (1)$$

where $\bar{x}$ is the sample mean, $\mu$ is the population mean, s is the standard deviation of the sample and N is the sample size, has a distribution similar to the normal distribution. Tables of "t" similar to those of the normal law have been prepared. Given a sample of size N, with mean $\bar{x}$ and standard deviation s, a table of Student's t can be used to find confidence limits for the mean $\mu$ of the population, on the assumption that x (in this case the test score) is normally distributed in the population.

Test 2. Confidence limits for the difference of the population means. It was found in Chapter V that the mean scores of the two samples of students were different on both the Algebra test and the General Test. It must

now be ascertained from the sample statistics whether the population means would differ significantly at a predetermined level of probability. The tables of Student's t can be used for this significance test also, the required formula being

$$t = \left[\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)\right] \Big/ \hat{\sigma}_{12} \qquad (2)$$

where $\bar{x}_1$ and $\bar{x}_2$ are the sample means, $\mu_1$ and $\mu_2$ are the population means, and $\hat{\sigma}_{12}^2$ is an unbiased estimate of the variance of the numerator, taken as a weighted mean of estimates from the two samples separately, weighted according to their respective degrees of freedom.

Test 2(a). The F-test. In testing the significance of the difference of the means of two populations by the use of formula (2), it is assumed that the populations from which the samples are drawn have the same variance. It is possible to find whether this assumption is justified, before proceeding with the significance test, by determining the significance of the ratio of the population variances. This is done by the so-called F-test. The ratio $\sigma_1^2 / \sigma_2^2$ is estimated from sample statistics; the significance of the ratio is then found in a table of F. Further detail on this test may be found in Appendix G, page 154.

# II. SIGNIFICANCE TESTS

## A. THE ALGEBRA TEST

Before carrying through the significance tests outlined above, preliminary calculations must be made to determine the values of the standard deviations $s_1$ and $s_2$ and the variances $s_1{}^2$ and $s_2{}^2$ of the samples, and of the quantity $\hat{\sigma}_{12}$ in formula (2).

### Standard Deviations of the Samples

The standard deviation of a distribution is the most important measure of dispersion. It is defined as the root-mean-square average of the deviations of the observed values from their mean, and is given by the formula

$$s_x = \left[\frac{1}{N}\sum_i f_i(x_i - \bar{x})^2\right]^{1/2}, \quad N = \sum_i f_i \qquad (6)^{42}$$

The square of the standard deviation is called the variance ($s_x{}^2$) of the distribution.

A re-arrangment of formula (6), together with certain

---

[42] A single sequence of formula numbers is used in the text and in Appendix G.

substitutions gives

$$Ns_x^2 = \sum f_i x_i^2 - \frac{1}{N}(\sum f_i x_i)^2 \qquad (8)$$

This is the usual form for the calculation of $s_x$ for a set of ungrouped values of x. However, if the sample is large, the values may be grouped, and a "coded" variate u introduced to simplify the arithmetic. A suitable fixed value $x_o$ (= the class mark for one of the central classes of the distribution) is subtracted from each class mark $x_i$; then if c is the class interval, $x_i = cu_i + x_o$. The $u_i$ are consecutive small integers when the classes are all equal in width. Formula (8) may accordingly be written in terms of the u variate, and the relation $s_x = cs_u$ employed to get the standard deviation of the distribution.

This procedure has been used to obtain the standard deviations of the 1938 and 1959 scores on the Algebra test and the General Test. The first part of the calculation is arranged in tabular form on the following page; the subscripts are omitted in the column headings. It may be mentioned that the column headed $f(u + 1)^2$ is included for the purpose of carrying out a "Charlier check" on the arithmetic.

Calculation of the standard deviation of the 1938 scores on the Algebra test, using the coded variate u.

| Class Interval | Class Mark x | Frequency f | u | fu | fu(u) $= fu^2$ | $f(u + 1)^2$ |
|---|---|---|---|---|---|---|
| 0-4 | 2 | 5 | -5 | -25 | 125 | 80 |
| 5-9 | 7 | 6 | -4 | -24 | 96 | 54 |
| 10-14 | 12 | 12 | -3 | -36 | 108 | 48 |
| 15-19 | 17 | 26 | -2 | -52 | 104 | 26 |
| 20-24 | 22 | 16 | -1 | -16 | 16 | 0 |
| 25-29 | 27 | 25 | 0 | 0 | 0 | 25 |
| 30-34 | 32 | 26 | 1 | 26 | 26 | 104 |
| 35-39 | 37 | 24 | 2 | 48 | 96 | 216 |
| 40-44 | 42 | 24 | 3 | 72 | 216 | 384 |
| 45-49 | 47 | 19 | 4 | 76 | 304 | 475 |
| 50-54 | 52 | 12 | 5 | 60 | 300 | 432 |
| 55-59 | 57 | 4 | 6 | 24 | 144 | 196 |
| 60-64 | 62 | 3 | 7 | 21 | 147 | 192 |
| 65-69 | 67 | 1 | 8 | 8 | 64 | 81 |
| 70-74 | 72 | 0 | 9 | 0 | 0 | 0 |
| | | 203 | | 182 | 1746 | 2313 |
| | | | | X2 | | |
| | | | | 364 | | |

Charlier check: $\sum f(u + 1)^2 = \sum fu^2 + 2\sum fu + \sum f$

Left hand side = 2313

Right hand side = 1746 + 364 + 203 = 2313

Substituting appropriately in formula (8), in terms of the u variate,

$$203 s_u^2 = 1746 - \frac{1}{203}(182)^2$$

$$= 1746 - \frac{33124}{203}$$

$$= 1746 - 163.17$$

$$= 1582.83.$$

Then $$s_u^2 = \frac{1582.83}{203} = 7.7972$$

$$s_u = (7.7972)^{\frac{1}{2}} = 2.792,$$

whence $$s_x = cs_u = 5(2.792) = 13.960.$$

Thus the standard deviation is 13.96 to four figures. The variance is $(13.96)^2 = 194.88$.

The detailed calculations of the standard deviations and variances of the 1959 scores on the Algebra test and of the 1938 and 1959 scores on the General Test may be found in Appendix H, page 158. The results, together with other data necessary for the tests of significance, are given in Table VI, page 80.

Tests of Significance: Calculations

1. Confidence limits for the population mean. It has been mentioned that the formula

$$t = \frac{\bar{x} - \mu}{s/(N - 1)^{\frac{1}{2}}}$$

can be used to find confidence limits for the mean $\mu$ of a population from sample statistics. A re-arrangement of the formula gives

TABLE VI

SAMPLE SIZE, MEAN RAW SCORE, STANDARD DEVIATION AND VARIANCE FOR THE ALGEBRA TEST, 1938 AND 1959

| Year | Sample size<br>N | Mean Raw Score<br>$\bar{x}$ | Standard deviation<br>$s_x$ | Variance<br>$s_x^2$ |
|---|---|---|---|---|
| 1938 | 203 | 31.36 | 13.96 | 194.88 |
| 1959 | 481 | 26.66 | 15.79 | 249.22 |

$$\mu = \bar{x} \pm \frac{ts}{(N - 1)^{\frac{1}{2}}}$$

For large samples, N - 1 may be replaced by N, and the proper value of t for the 99 per cent confidence limits is 2.576.

The claculation with respect to the 1938 Algebra test scores follows. The sample statistics are: N = 203, $\bar{x}$ = 31.36, s = 13.96. Substituting these, together with the appropriate value of t, we have

$$\mu = 31.36 \pm \frac{2.576(13.96)}{(202)^{1/2}}$$

$$= 31.36 \pm 2.53$$

$$= 28.83 \text{ or } 33.89.$$

There is, therefore, a 99 per cent chance that the population mean lies between 28.83 and 33.89. From this it may be concluded that had the entire population taken the test the mean raw score would (with a probability of 99 per cent) have differed from the mean raw score of the sample by not more than two and one half marks.

For the 1959 sample the corresponding statistics are: sample size $N = 481$, mean raw score $\bar{x} = 26.66$, standard deviation $s = 15.79$. For this population, then,

$$\mu = 26.66 \pm \frac{2.576(15.79)}{(480)^{1/2}}$$

$$= 26.66 \pm 1.86$$

$$= 24.80 \text{ or } 28.52.$$

There is thus a 99 per cent chance that the population mean would differ from the sample mean by less than two marks.

2. Confidence limits for the difference of means. It has been mentioned that the particular significance

test for the difference of the population means which is used in this study assumes that the samples employed are drawn from populations having the same variance. This assumption may be checked by an F-test, which measures the significance of the ratio of the population variances. For samples as large as those involved here, it is permissible to use the sample variances as estimates of the population variances. Hence, F = 249.22/194.88 = 1.28. From a table of the distribution of F, the 5 per cent point is 1.22 and the 1 per cent point is 1.33. The F tables are set up for one-tailed tests since in practice it is usual to expect one population variance to exceed the other, that is, to expect the ratio $\sigma_1^2/\sigma_2^2$ to be greater or less than 1. If there is no reason for believing that either variance is greater than the other, a two-tailed test of the null hypothesis is required, since the hypothesis will have to be rejected both when F is too great and when F is too small. For a two-tailed test, then, the 5 per cent point really corresponds to a 10 per cent level of significance and the 1 per cent point to a 2 per cent level. Consequently, in the present case the ratio is significant at a level between 10 per cent and 2 per cent, or, more specifically, about 6 per cent. Thus there is a probability of about 6 per cent of obtaining

by pure chance sample variances as different as these. The null hypothesis that the population variances are not different may therefore be accepted quite reasonably. Confidence limits for the difference of the means may now be determined.

Formula (2), page 75, may be re-arranged to give

$$\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2 - t\hat{\sigma}_{12} .$$

Taking the sample statistics of the Algebra test from Table VI, we have

$$\hat{\sigma}_{12}^2 = \frac{481(15.79)^2 + 203(13.96)^2}{480 + 202} \cdot \frac{481 + 203}{(481)(203)}$$

$$= 1.633.$$

Then $\hat{\sigma}_{12} = \pm\sqrt{1.633} = \pm 1.278.$

For the 1 per cent confidence limits the table of t gives t = 2.576 for a two-tailed test. At this level of confidence, then, $\mu_1 - \mu_2 = 31.34 - 26.66 \pm 2.576(1.278)$

$$= 4.68 \pm 3.29.$$

The 1 per cent confidence limits for the difference of the population means are therefore 1.39 and 7.97. Since these limits do not include zero, the difference of the means is significant at the 1 per cent level.

## B. THE GENERAL TEST

The data from the General Test have been treated in the same way as those from the Algebra test. The detailed calculations are given in Appendix H, page 158. The sample statistics are shown in Table VII below, followed by the results of the tests of significance.

TABLE VII

SAMPLE SIZE, MEAN RAW SCORE, STANDARD DEVIATION AND VARIANCE FOR THE GENERAL TEST, 1938 AND 1959

| Year | Sample size $N$ | Mean raw score $\bar{x}$ | Standard deviation $s_x$ | Variance $s_x^2$ |
|---|---|---|---|---|
| 1938 | 203 | 56.49 | 9.410 | 88.55 |
| 1959 | 466[ø] | 60.78 | 11.76 | 138.3 |

[ø] The number of students in attendance for the General Test was smaller than that for the Algebra test. This may have been because in some of the schools the Algebra test was given in the morning, and a dislike of tests may have caused some students to stay away in the afternoon when the General Test was scheduled.

The 99 per cent confidence limits for the population means are: for the 1938 population, 54.79 and 58.19; for the 1959 population, 59.38 and 62.18. This

is a deviation of ± 1.70 from the 1938 sample mean, and of ± 1.40 from the 1959 sample mean.

The F-test applied to the sample variances resulted in a ratio of 1.56, which indicates, at a level of significance below 2 per cent, that the variances of the 1938 and 1959 populations are the same.

Finally, the confidence limits for the difference of the population means are 1.89 and 6.69. These limits do not contain zero, and since the test was carried out at the 1 per cent level, the difference is highly significant.

## III. CONCLUSION

The statistical tests carried out in this chapter show that the difference of the population means on the Algebra test as calculated from sample statistics is highly significant. It is therefore necessary to reject the hypothesis that there is no difference in competence in Algebra between the Grade IX students of 1959 and those of 1938 in the Edmonton public school system. The tests indicate that the students' proficiency in Algebra in

1959, as measured by the 1938 Departmental Examination, is significantly inferior to that obtaining in 1938.

# CHAPTER VII

## FACTORS AFFECTING THE RESULTS

There are about a dozen factors which logically might be expected to have a differentiating effect on the test scores of the two groups of pupils which compose the subject of this study. These may be divided into two categories: those which are measurable to some degree, and those which are not subject to any precise quantitative appraisal. These are considered separately, and an assessment is made of their net effect.

### Factors Which Are Measurable in Some Degree

The measurable factors are set forth in Table VIII on the following page. The effect (positive or negative) that the differences would be expected to have on the scores of the 1959 pupils is indicated in the last column.

*Comment*. The I.Q. scores from which the means appearing in Table VIII were derived were obtained from the Grade IX record cards at the Department of Education. The scores of the 1959 sample were based for the most part on the Laycock test. Of the 1938 sample, nearly all individuals were rated by both the Laycock test and the Willis-

## TABLE VIII

## FACTORS, MEASURABLE IN SOME DEGREE, WHICH WOULD BE EXPECTED TO INFLUENCE THE TEST SCORES OF THE TWO SAMPLES

PART A

| Item | 1938 sample | 1959 sample | Expected effect |
|---|---|---|---|
| Mean I.Q | 102 | 109 | + |
| Mean age | 15 y. 4 m. | 15 y. 1 m. | |
| Mean attendance | 173 days | 188 days | + |
| Mean class enrolment | 37 | 29 | + |

PART B

QUALIFICATIONS OF TEACHERS

| Item | 1938 | 1959 | |
|---|---|---|---|
| Number of teachers teaching Grade IX mathematics | 11 (all schools) | 12 (sample schools) | |
| B.A., B.Sc. or B.Ed. degree | 4 | 9 | |
| B.Ed. and a second bachelor's degree | 0 | 2 | |
| M.A. | 1 | 0 | ±? |
| B.A., M.Sc. | 1 | 0 | |
| No degree | 5 | 1 | |
| First Class or Professional certificate | 11 | 10 | |
| Junior E (Perm.); Standard S (Interim) | 0 | 2 | |

Smith test. Since the Laycock score was as often above as below the Willis-Smith score, each pupil's I.Q. was taken as the mean of the two recorded scores. The I.Qs. for one of the 1938 schools were based on the Terman test.

The figures in Table VIII indicate that the 1959 pupils were brighter than the pupils of 1938. This has already been implied by the scores on the General Test. However, there is a reason for believing that the difference in intelligence was less than the figures suggest. Among the record cards for both samples there were some[43] which carried no I.Q. score--perhaps because the pupils were absent at the time of the test or because they were immigrants for whom no scores had been forwarded. The mean raw score on the Algebra test was computed for the members of each sample whose I.Qs. were not on record. In the case of the 1959 group, this score was 8 per cent below the sample mean, but for the 1938 group it was 1.6 per cent above the sample mean. The same inquiry was made with respect to the General Test; in this case the 1959 group was 5.1 per cent below the sample mean, while the 1938 group was 2.2 per cent above the sample mean. It may be inferred that had the I.Qs. of these pupils been

---

[43] Thirty-two in 1959, nine in 1938.

available, their inclusion would have lowered the mean intelligence rating of the 1959 sample and raised that of the 1938 sample, thus diminishing the difference.

## Factors Not Subject to Precise Quantitative Assessment

Algebra Test Question No. 20. It might be supposed that the retention of Question No. 20 on the Algebra test for the 1959 pupils would have an appreciable negative effect on the mean score of this group, since the operation required was not dealt with in their course. An examination of this possibility shows, however, that the effect was of no significance.

The operations of subtraction and division, being less basic in number experience than addition and multiplication, and learned subsequently to them, are in general performed with less facility and accuracy. Thus it is reasonable to suppose that even if the 1959 pupils had been taught algebraic long division, they would have scored more poorly on Question 20 than they did on Question 13, which required the inverse operation, multiplication of a trinomial by a binomial. Table V, page 71, shows that the group as a whole earned approximately 260 more marks on Question 13 than on Question 20. Presumably they would have earned fewer than 260 additional marks

on Question 20 if they had been prepared for it. The net gain in mean score, distributing this hypothetical addendum among the 481 individuals, would be about one half mark per pupil--scarcely enough to affect the significance of the difference of the sample means.

There is another point which deserves mention in this connection. *The Programme of Studies for The Intermediate School* (effective in 1938), in the section headed "Regulations Governing the Grade IX Examination", sets out the scoring policy with respect to Language usage on all examinations:

> *The final value assigned to a candidate's answer papers in Literature and Language will depend not only on their content, but on the quality of the written expression in respect to spelling, clearness, correctness and good form.*
>
> The same treatment will be given to the candidate's answer papers in *Social Studies*, *Mathematics*, *Science and Health Education*. On every paper a definite portion of the score will be assigned for spelling, clearness, correctness and good form.[44]

According to an official of the Examinations Branch, the usual method of carrying out this policy in the case

---

[44] *Programme of Studies for The Intermediate School and Departmental Regulations Relating to the Grade IX Examination* (Edmonton: A. Shnitka, King's Printer, 1935), p. 14. The italics occur in the original.

of the examination in mathematics was to deduct up to five marks from the student's total score for errors in spelling and English usage.

No such deduction was made in scoring the 1959 papers. Hence in respect of any change of marks which might have resulted from Language errors, the 1959 scores are, if anything, higher than if the papers had been scored in 1938.

Motivation. It might be presumed that the 1938 students were the more intensely motivated during the writing of the Algebra test, since for them it was the final examination on which they were to pass or fail their year's work. How much difference this could make in their scores is, however, an open question. A strong desire to succeed may increase a pupil's intellectual effort, but its effect in a test situation is limited since it can neither call up facts which were never learned nor magically produce skills not previously developed by practice. Motivation certainly generates results during the learning process. At examination time it is a stimulus, but it cannot compensate for ignorance. It seems safe to say that if a stronger motivation in the 1938 students was a significant factor in producing a higher mean score in

Algebra, it should have produced a higher score in the General Test as well. Such was not the case. Thus it would be a mistake to ascribe much potency to this factor.

"Last-minute" review. Another circumstance which might be suspected of contributing to the higher mean score of the 1938 sample is the advantage gained by them from the final few days' intensive review which was not permitted the comparative group. The 1959 pupils wrote the test about the 12th of June, which was more than a week earlier than the date of the Departmental examination in 1938. The question is: by how many points would a week's review raise the mean score of a group of pupils in Grade IX Algebra? It is impossible to answer this. Indeed, chance would play a part, in that certain points of emphasis in the review might or might not correspond to sections of the course pre-destined (so to speak) for testing. There are, however, two points to consider in estimating the effect of the review factor. The first is that in all likelihood not more than half of the extra week's classes in 1938 were devoted to strengthening the Algebra; the other half would be spent on Geometry. The second is drawn from Table VIII, page 88, which shows the mean year's attendance in 1938 as 173 days, compared with 188 days in 1959. It must be acknowledged that this

increment did not give the 1959 pupils a clear advantage of fifteen additional days' instruction, since about 11 days of their total attendance occurred _after_ they wrote the Algebra test. Nevertheless, their 177 days' pre-test attendance represent about nine days more instruction time than was received by the 1938 pupils from whose total of 173 days must be subtracted the last five non-instructional examination days in June. This means that the effective length of the school year in 1938 was nearly two weeks less than in 1959. It is submitted that this loss of normal class time would not be made up by the concentrated cramming of the final week's review.

_Change of emphasis in the teaching of mathematics_. On the basis of the aims and objectives of mathematics teaching as set forth in the 1938 and 1959 curriculum guides it cannot be said that there was any change in philosophy or emphasis from the earlier to the later period. Officially the stress in 1938 was on a wide range of interpretative mathematics, and explicitly _not_ on techniques of computation. In 1959, the development of habits of logical thinking and of the ability to analyse problems is placed ahead of the mechanical operations required to obtain solutions. In neither guide is computational skill entirely ignored, however. In 1938 attention

was directed to the many sets of exercises in the Grade VII and VIII textbooks designed for review and drill on the fundamental skills; and in 1959 the teacher was reminded that "the computational part of mathematics constitutes one of the most important skills required in modern life."[45]

None the less it is possible and even likely that *in practice* there was a difference in emphasis. There are two arguments to support this contention. Firstly, the idea of subordinating manipulative and computational skills to the mathematical interpretation of the environment, the grasping of broad generalizations, and the understanding of relationships was relatively new in 1938. The pupils had been accustomed in the elementary school to regarding speed and accuracy in calculation as a desirable goal, and to the use of drill to attain it; the teachers also had been bred in the same tradition, possibly to an even greater degree. By 1959, however, the importance of acquiring meaningful concepts and of appreciating relationships had become established and familiar, especially, perhaps, to younger teachers who had themselves been pupils

---

[45] *Junior High School Curriculum Guide for Mathematics* (Province of Alberta, Department of Education, 1952), p. 3.

during the two-decade interval. Thus, while the stated aims and objectives of mathematics teaching had not changed, it seems reasonable to believe that their effect in practice would be stronger in the later part of the period than in the earlier.

Secondly, notwithstanding the official declared shift in emphasis from manipulative skill to basic insights and appreciations, the teacher of 1938 was equipped in two ways to maintain the obsolescent tradition: by his own training, and by a textbook which implicitly stressed manipulation to a degree which was inconsistent with the doctrine of the curriculum. The 1959 teacher, on the other hand, was better prepared to implement the philosophy of mathematical awareness through his personal learning experience and by means of a more appropriate textbook and a detailed and carefully-prepared curriculum guide.

Consequently it seems altogether likely that the Grade IX pupils of 1938 were influenced by a more rigorous attitude toward manipulative proficiency in mathematics than were their scholarly posterity of 1959, and that the latter received a more deliberate instruction in basic meanings and relationships.

Departmentalization of instruction. The writer has been told by Mr. R. Shaul, who was teaching Grade IX in Edmonton in 1938, and who is now Supervisor of Junior High Schools for the Edmonton Public School Board, that the degree of departmentalization of instruction in the city public schools was probably greater in 1938 than in 1959. This was because the present "blocking" system[46] was not then in use, and teachers specialized more narrowly in their chosen subject areas. However, since a large degree of specialization still exists in the blocked areas of instruction, a difference in the effectiveness of teaching resulting from this factor is not to be presumed.

Relative selectness of the population. It is often said that today's school population is less select than that of twenty or more years ago because the drop-out rate is lower now than it used to be. Nothing has come out of the present study to support this contention with respect to the Grade IX level for the period considered. In the Edmonton public school system the net loss of Grade IX pupils in 1937-38 was thirty-five, which represented 3.7 per cent of the population. In 1958-59 the net

---

[46] A system which combines in pairs or groups subjects in which instruction can be effectively integrated; e.g., Mathematics-Science, Social Studies-Language. Instruction is departmentalized by blocks.

loss was sixty-two, which was 4.1 per cent of the population. These figures state the difference between the October and June enrolments, and include transfers as well as drop-outs; but unless there has been a marked increase in emigration (the contrary, in fact, is the case) the drop-out rate has not decreased.

If together with this apparent stability the fact is considered that the mean I.Q. of the Grade IX population (inferred from that of the sample) was if anything higher in 1959 than in 1938, it must be concluded that the pupils of 1959 were not a less select group than those of twenty-one years earlier.

Other factors. There are still other factors which might have had an effect on the 1959 scores. For instance, Again, a change in the attitude of teachers and parents toward homework could have resulted in an increase or a decrease in the amount of extra-school practice in the skills of mathematics. Whether parents' and teachers' views on the importance of homework have in fact changed is not known by the writer. One could not be sure that such a change had come about, nor would its consequences be assessable, unless it had derived from an explicit and uniformly-applied policy by school authorities. No

[illegible] a diff[illegible] in the [illegible] writing of it [illegible] of 1959 to [illegible] classes. [illegible] in time [illegible]; but this [illegible] be [illegible]

directive in this connection has been issued by the Edmonton Public School Board. There is, however, a mild declaration of policy in the Department of Education Junior High School Handbook:

> While some homework outside of regular school hours will be necessary, more time and better opportunity for well-directed, intensive study is provided under the block system. Thus it is hoped that the assignment of homework for junior high schools need not be too burdensome.[47]

This suggests that the recent and present tendency is away from prescribed home study.

Another question more easily raised than settled is whether the generally improved economic conditions of the post-war years have made students (1) more or less amenable to discipline and counsel, (2) more or less appreciative of the value of schooling, (3) more or less keen to learn than were their parents. Again, what effect, if any, develops in pupil achievement as a result of more attractive school buildings, better lighting and improved facilities? Finally, what educational net gain or loss may be ascribed to the subtle changes in social standards

---

[47] Junior High School Handbook (Edmonton: Province of Alberta, Department of Education. L. S. Wall, Queen's Printer for Alberta, 1955).

and values brought about by successive periods of depression, war, and post-war prosperity; by incredible technological advances which continuously remind the rising generation of the need for more and better training; and by the ever-present state of international tension, which represents the surface of a vast and growing sea of human problems? Undoubtedly these things influence children and indirectly affect school achievement. To measure their effects both qualitatively and quantitatively is a task for future researchers.

Summary. Of the five measurable factors in Table VIII (p. 88), three (I.Q., attendance, class size) would be expected to cause the scores of the 1959 sample to be higher than those of the 1938 sample; the others (pupil age, and qualifications of teachers) should be judged indeterminate. An additional consideration favorable to the 1959 group is a superior ability as indicated by their higher General Test score. With respect to the factors for which no precise measurement is possible, it is pointed out that: (1) the inclusion of Algebra test Question No. 20 placed the 1959 pupils at no important disadvantage; (2) there is no reason to attribute the higher Algebra score of the 1938 sample to a stronger motivation, since this stimulus would have had a like

effect (noticeably absent) on the General Test score; (3) any advantage gained by the 1938 pupils from their final year-end review would be offset by the benefit of the nine additional days' instruction enjoyed by the 1959 pupils; (4) there was probably, in practice, a stronger bias toward manipulative proficiency in the earlier part of the period and toward comprehension, insight and appreciation in the later part; (5) there was a somewhat more concentrated specialization of instruction in 1938 than in 1959; (6) there is no evidence that the Grade IX population of 1959 was less select than that of 1938; (7) certain socio-economic influences might have operated either to the advantage or to the disadvantage of the 1959 pupils; but the effect of these is, for this study at least, impossible to evaluate.

The writer feels that those of the above factors which were advantageous to the 1959 pupils outweigh those which were unfavorable to them in their comparison with the other group. These pupils should, therefore, have done better on the Algebra test than did their predecessors of twenty-one years earlier.

# CHAPTER VIII

# SUMMARY AND CONCLUSIONS

## I. SUMMARY

The flood of criticism which has been directed at public education in this country and the United States during the last ten or twelve years has made advisable the carrying out of objective and controlled studies in carefully defined areas so that it might be claimed with confidence either that certain standards have declined or that they have been maintained. The present study is an attempt to measure the competence in Algebra of today's Grade IX pupils against the standard of twenty-one years ago. Its basic hypothesis is that there is no difference between the two generations with respect to competence in Algebra.

The majority of studies of a similar nature which have been made in Canada and the United States have shown that, in the areas investigated, the modern student is as good as or better than his prototype of a quarter century or so ago.

Since 1952 there has been in Alberta an attempt by the Department of Education to provide more guidance and assistance to teachers through the official curriculum guides and by the selection of more appropriate textbooks. Today's teacher of Grade IX Mathematics has at his disposal a more extensive body of advice and instruction for teaching than was available to teachers in 1938.

On June 12, 1959, a sample of 481 Grade IX students in six Edmonton public schools was given the Departmental examination in Algebra for 1938, and also the General Test of the same year. The sample was chosen with attention to considerations of practicability of procedure and representativeness of selection. A comparison sample of 203 students of the year 1938 was drawn at random from the record cards in the Department's files. The two samples were compared with respect to their test scores and with respect to other factors which would be expected to determine their achievement in Algebra as measured by the test.

The mean raw scores of the 1959 and 1938 samples were 26.66 and 31.36 respectively on the Algebra test and 60.78 and 56.49 respectively on the General Test. This indicated that the 1959 pupils were brighter than

those of 1938, but that they were less proficient in algebra. The differences in mean scores were subjected to statistical tests and were found to be highly significant. The null hypothesis which was set up at the commencement of the study was therefore rejected.

Consideration was given to a number of factors which would seem most likely to determine the pupils' level of competence in algebra. The factors evaluated were the mean I.Q., chronological age, number of days' attendance, and class enrolment for each sample; qualifications of teachers, certain peculiarities of the test itself, pupil motivation, the effect of year-end review, change of emphasis in the teaching of mathematics, departmentalization of instruction, and relative selectness of the populations. The net effect of these factors appeared to favor the 1959 group. These pupils should, therefore, have turned in the better performance on the test. Since they failed to do so, an explanation must be sought for their shortcoming.

## II. CONCLUSIONS

1. There has been a decline in proficiency in algebra at the Grade IX level in the Edmonton public

schools since 1938. The decline does not seem to be of an alarming magnitude when the absolute achievements of the two samples are compared: it is of the order of five per cent. A more valid comparison might be realized, however, by using not the mean score actually made by the 1959 sample, but an estimate of what they might reasonably be expected to have made (relative to the 1938 sample) considering their superior ability, the smaller pupil-teacher ratio in their classrooms, and their more regular school attendance. If this were done, the decline would appear more serious.

2. The present level of proficiency in algebra in Grade IX is regrettably low. This conclusion is drawn with reference to the 1938 algebra examination as a measuring instrument, and also with reference to the present algebra curriculum, which differs little, topically, from that of 1938. The writer bases the verdict on the fact that the mean score of the 481 pupils who wrote the test in 1959 was 36 per cent; and also on the contents of Table V, page 71, which show that the most frequently-occurring score for the majority of the test questions was zero.

By 1959 the prescribed bias in mathematics teaching--toward developing the ability to analyse problems,

to make correct interpretations and to perceive relationships--had become a tradition of two decades' standing. It would therefore seem logical to expect the 1959 pupils to perform especially well on the test questions which measure these abilities. But Table V does not show a more creditable record on the questions requiring mainly analysis and interpretation than on the manipulative types. There is here no indication of a close approach to the primary goals of the curriculum.

## III. INTERPRETATION

A study of this kind cannot be considered complete unless it suggests a cause of the condition which it claims to have discovered. It has already been argued that the decline in mathematical competence which this study reveals is not a result of a difference in the nature of the samples, nor in course content, nor in scoring standard, nor in the general ability of the two samples of students, nor in teachers' qualifications, nor in teacher-pupil ratio, nor in school attendance, nor in pupil motivation, nor in selectness of population. It is therefore necessary to look elsewhere for an explanation.

The Report of the Royal Commission on Education in Alberta expresses the view that certain community attitudes which have developed mainly since the war have had harmful effects on the disposition of young people toward education. On pages 41 and 42 the Report states:

> No small part of a disinclination toward education may be traced to out-of-school influences--the home and society at large. In an era of affluence . . . individuals have become enamoured of immediate personal benefits and self-interests. . . A long series of undesirable "public" practices and attitudes are put on exhibition to school youth. . . Many young people who leave school prematurely have succumbed to influence directed at them while they were still in school. There is evidence that many complaints from employers originate in this group, where achievement is grade X or below and whose attitudes partially reflect those of society.

This is a carefully considered criticism of the environmental situation now obtaining, and is worthy of serious attention. The present writer feels, however, that with specific regard to the Grade IX mathematics program, criticism may justifiably be directed at another fundamental factor, namely, certain points of official philosophy and policy underlying the curriculum and its implementation.

The thesis is accordingly submitted that an important cause of the decline and the present deficiency in algebraic competence at the Grade IX level is to

be found in the philosophy and the declared aims appearing in the curriculum guides which have been in use since 1936. This philosophy with its associated aims is, in the opinion of the writer, unrealistic in its emphasis. It is expressly stated in the curriculum guides that: (1) it is not desirable to spend time on perfecting techniques of computation which the students will seldom if ever use; (2) that the aim of the course is to interpret the environment rather than to give skill in computation; (3) that the stress is to be placed throughout upon wide training rather than on the mastery of difficult techniques; (4) that the course should develop in the pupils the ability to think clearly and to use information, concepts and general principles (as well as fundamental skills); (5) that the course should develop desirable attitudes and a wide range of interest and appreciation; (6) that to solve a few problems which tend to develop logical thinking is preferable to solving many problems where the only goal is a "right answer"; (7) that the analysis of a problem is more important than the necessary mechanical operations required in the solution.

These seven principles may be classified under two headings:

1. Learnings which are relatively unimportant

(computational techniques and skills);

2. Learnings which are of prime desirability (the ability to think, to analyse problems and to apply generalizations; the development of attitudes, appreciations and interests).

The writer questions the point of view of the curriculum guides as to the application of these principles at the junior high school level.

Grade IX is one of the natural disembarking points in the school system. Many pupils who have low academic ability leave school at the end of their Grade IX year, some because they fail, others because they have reached the legal school-leaving age and see little reason to continue the uninteresting struggle. Their limited qualifications for employment restrict them to routine types of jobs, and force them to settle in an elementary occupational environment. Here they require not the generalizations but the specifics of mathematics: the ability accurately to count, to measure, to calculate, to work out correct answers. Their personal need of mathematics is confined to arithmetic, and the greatest asset in its use is computational proficiency. Thus the best service the mathematics course can render these pupils is to equip

them with a maximum degree of computational skill, provided through concrete situations related to the out-of-school environment. It is unrealistic to emphasize for these pupils the subtleties of generalizations, interrelationships and analysis, for they have limited ability to comprehend them.

The preceding paragraph does not imply that the mathematics course should be designed for the lowest stratum of student ability. Certainly there must be stimulation and challenge in it for the better students. Yet even for these the objective of manipulative skill is desirable; such skill is never superfluous in mathematics. The majority of the abler group intend, upon entering high school, to spend at least three more years as students, and to acquire a high school diploma or matriculation standing. This is the level, with respect to both subject matter and student maturity, where a serious study of the general methods of algebra may be introduced; and some expectation held that the student may begin to grasp the meaning of logic and the power of analysis, and to appreciate the rôle of mathematics in the modern world. Even here a beginning only can be made, for deep insights and appreciations do not come to the immature in an abstract science like mathematics; these things are among the last outcomes

of any study, never among the first. Indeed, to some extent they are incidental learnings, the higher derivatives of factual knowledge and practical use.

In discussing further the last four of the principles listed above (taken mainly from the current mathematics curriculum guide), it is acknowledged that they are worthwhile aims. That it is highly desirable to teach students to think, to analyse problem situations, to generalize, to discover fundamental relationships and to develop appreciations is accepted and endorsed without reservation. The question is to what extent these things can be taught, if indeed they can be taught at all; and whether the present curriculum is suited in an adequate degree to the cultivation of them.

For one thing, it is quite unnecessary even today for the layman to be versed in the broad generalizations of mathematics in the same sense that he should be informed on the principles and progress of science. Newspapers and popular magazines daily carry articles which require the reader to have a smattering of physics, chemistry, geology, biology and astronomy. But one seldom hears any direct reference to mathematics, its specialties, its applications or its continuing development. This is because

mathematics is a kind of backstage resource. It forms much of the theoretical basis of science, but its uses and effects are concealed behind or within that which is visible and palpable. The achievements of science are everywhere evident, and the work of the scientist is continually publicized and even glamorized; but the work and achievements of the mathematician are relatively unknown. It is desirable for everyone to understand in a general way something of the structure of the universe, of the atom and of living matter, but it is unnecessary for many to be familiar with the structure of a binomial expansion or a mathematical matrix.

Another question should be raised. The curriculum guides place much emphasis on the development of analytical power and logical thinking. There are probably no abilities which are more to be desired than these as an outcome of the mathematics program. But is it in fact possible to teach people to think? It is common school practice to illustrate problem solving by the use of more or less stereotyped solutions to typical (and often artificial) mathematical problems. Students learn to solve variations of the same "problems" by means of the same procedures. But this is no more than learning a standard method of solving a stock problem--in other words, it is

just a higher stage in the acquisition of manipulative skill. True analytical ability consists in recognizing an entirely new situation as a synthesis of familiar elements. The exact nature of this ability and the question of the degree to which it can be developed is the subject of a rather extensive literature.

Dewey, Wertheimer, Vinacke, Fehr, Georges, Henderson and Pingry[48] are among the writers who have attempted to discover and describe how the mind attacks and solves a problem. The most thoroughly experimental approach to this question so far made was that of Buswell,[49] who tried to discover whether the problem-solving mind follows any discernible patterns of thought. Buswell's purpose, in

---

[48] John Dewey, How We Think (New York: D. C Heath and Company, 1933); Max Wertheimer, Productive Thinking (New York: Harper & Brothers, 1945); W. Edgar Vinacke, The Psychology of Thinking (New York: The McGraw Hill Book Company, 1952); Howard E. Fehr, "The Role of Insight in the Learning of Mathematics," The Mathematics Teacher, October, 1954, pp. 386-392; J. S. Georges, "Learning to Solve Problems Intelligently," School Science and Mathematics, December, 1956, pp. 701-707; Kenneth B Henerson and Robert E. Pingry, "Problem Solving in Mathematics," Chapter 8 of The Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1953).

[49] Guy T. Buswell, "Patterns of Thinking in Solving Problems," University of California Publications in Education, Vol. 12, No. 2, pp. 63-148 (Berkely: University of California Press, 1956).

his own words, was "to find whether there are general patterns of problem solving thinking which characterize sizeable groups of individuals, or whether the thinking process is so varied in character that only descriptions of individual thinking can be given."[50]

Buswell constructed tests especially for his study, and administered them to 209 high school students and 290 college students. The main data derived from the experiment were scores on the written portion of the tests and voice recordings made while some of the subjects "thought aloud" as they worked out the problems. One of the several techniques used was a system in which the subject mapped his own progress through the solution by accepting or rejecting suggestions printed on a sequence of cards.

Here are Buswell's main discoveries:

1. In certain problems which were eminently suited to algebraic methods of solution, the great majority of the subjects studied by the tape recording system used a non-algebraic approach. They seemed spontaneously to adopt their own methods of logical analysis, and gave the

---

[50] Ibid., p. 63.

impression that they felt insecure in the algebraic medium.

2. Some of those studied by the card method used a random trial and error approach. This seemed to indicate a stronger tendency to experiment with the more promising of several ready-made routes than to make an independent analysis.

3. Those who solved the problems successfully used a wide variety of patterns of thought.

Buswell concluded that either great variations in the process of problem solving are normal and are to be accepted, or schools have been ineffective in teaching a technique of problem solving. He comments: "The evidence gives no support to the notion that problem solving must follow the neat, precise recipes that are frequently encountered in textbooks on methodology."[51]

One implication of Buswell's findings for teaching seems to be that since patterns of problem-solving thinking cannot be classified or systematized, it is not possible to teach the basic analytical processes and tactics

---

[51] Ibid., p. 133.

of insight which are the essence of problem solving. The teacher can, however, do much to develop habits of careful reading and of generalizing; he can demonstrate fruitful methods of attack on problems and give training in the required skills; and he can stimulate and encourage his students' interest and efforts.

To sum up, the present writer is not arguing that an understanding of the relationships and generalizations of mathematics should be abandoned as a teaching objective at the junior level, but that a larger share of emphasis be restored to the acquisition of the practical computational skills. Support for this view may be cited from authoritative sources. For example, the Joint Commission of The Mathematical Association of America and The National Council of Teachers of Mathematics stated a number of years ago that "The paramount mathematical need of the average citizen is for a greater knowledge of arithmetic than is now common"--arithmetic being interpreted broadly, with emphasis on the citizen's need for familiarity with a wide variety of practical applications.[52] The Report of

---

[52] A fuller statement of this claim may be found in a passage quoted on page 2 of the Senior High School Curriculum Guide for Mathematics issued by the Department of Education, Province of Alberta, September, 1959.

*The Royal Commission on Education in Alberta* offers as Recommendation No. 63: "That general facility in arithmetical skills be re-emphasized with particular regard for accuracy and automatic response." This recommendation was made with reference to the elementary grades, but the *Report* points out that such renewed emphasis "would not be incompatible with other aspects of the program at all school levels."[53]

Again, the present writer's argument is not against the development of insights and appreciations; rather it is that the possibilities of this kind of outcome in mathematics are so severely limited by the immaturity and rudimentary knowledge of the younger adolescent that it is unrealistic to give these things precedence over basic knowledge and computational proficiency in the aims of the junior high school. Finally, the argument is not against the desirability of enlarging analytical power and logical thinking; it is merely against believing or pretending that a great deal can be done in this respect when in fact there is no evidence that much can be done at all.

---

[53] *Op. cit.*, p. 108.

The writer is aware that in expressing his opinion as to a contributory cause of the decline in the pupils' algebraic proficiency he has re-entered the realm of hypothesis--which is the starting point for further research. It is suggested that a possibly worthwhile follow-up inquiry could be a study of scholastic standards--the values, utilities and conditions that cause them to change and by which they are judged good or bad.

## FINALE

This study makes no claim to complete rigor or exhaustiveness. The design and procedure were prescribed by circumstances rather than set up in conformity with ideals. The argument, by parts and in total, is not unchallengeable. Yet the study has measured such pertinent factors as are measurable, and it has attempted logically to evaluate others. Within the limitations of the measuring instruments and methods and of the writer's judgment, it has indicated that the Grade IX pupils of the Edmonton public school system are underachieving in algebra, or rather that the system is underachieving with respect to its output of algebraic understandings and skills at the Grade IX level.

In 1959 at least, according to the study, the pupils fell short of the standard of algebraic competence which obtained in their parents' day, and this in spite of certain advantages pertaining to their learning situation. This decline may well have occurred in the province as a whole, since provincial examination marks are year by year consistently lower than those of the cities.[54]

The results of the study suggest a re-examination of certain aspects of the mathematics program at the elementary and junior high school levels. With respect to the aims and objectives of the curriculum, the writer advocates a stronger emphasis on the manipulative and computational skills, with, of course, due attention to the understanding of the supporting principles. Without such emphasis it is difficult to see how genuine competence, which includes both understanding and skill, can be attained. Whitehead has said that " . . . education should turn out the pupil with something he knows well and something he can do well."[55] In mathematics, the

---

[54] See the Annual Reports of the Department of Education.

[55] Alfred North Whitehead, The Aims of Education (New York: The Macmillan Company, 1929. Reprinted by The New American Library as a Mentor Book, 1949).

pupil should both know and be able to do. The principles are vital, but the ultimate test is a test of performance.

To close this study the following recommendation by Partlow is offered:

> . . . in the United States approximately 100 times as much money is spent on industrial research as on research in the humanities, the arts and the social sciences combined . . .
>
> . . . it is suggested that the larger cities . . . or the Department of Education, or both, might assist education . . . by establishing departments of research whose responsibility it would be to assess continually different phases of the educational programme and to discover what distribution of emphasis is best in the various subject areas. Their findings should be published to provide information to an interested public in a democracy. Adequate research of this type would help to take the guess-work out of education, and to promote the educational progress which our human resources deserve.[56]

---

[56] *Op. cit.*, p. 141.

# BIBLIOGRAPHY

# BIBLIOGRAPHY

## A. BOOKS AND ARTICLES FOR THE LAY READER, CRITICAL AND OTHERWISE, DEALING WITH MODERN EDUCATION

### 1. Books

### Canadian Publications

Humble, A. The Crisis in Canadian Education. Toronto: The Ryerson Press, 1959.

Hume, William E. and Taylor, Harold F. Trouble in the School. Bracebridge (Ontario): Bracebridge Books, 1958.

Katz, Joseph. Canadian Education Today. New York: The McGraw Hill Book Company, Inc., 1956.

Neatby, Hilda. So Little For The Mind. Toronto: Clarke, Irwin, 1953.

Neatby, Hilda. A Temperate Dispute. Toronto: Clarke, Irwin, 1954.

St. John, J. Spotlight on Canadian Education: background to the Canadian Conference on Education, 1958. Toronto: W. J. Gage Company, 1959.

### American Publications.

Adler, Irving. What We Want In Our Schools. New York: The John Day Company, 1957.

Adler, Mortimer J. and Mayer, M. The Revolution in Education. Chicago: University of Chicago Press, 1958.

Bell, Bernard I. Crisis in Education. New York: The McGraw Hill Book Company, Inc., 1949.

Bestor, Arthur. The Restoration of Learning. New York: Alfred Knopf, Inc., 1955.

Cary, Sturges F. (ed.). New Challenge To Our Schools. New York: The H. W. Wilson Company, 1953.

Conant, James Bryant. The American High School Today. New York: The McGraw Hill Book Company, Inc., 1959.

Dunn, Joan. Retreat From Learning. New York: David McKay Company, Inc., 1955.

Fine, Benjamin. Our Children Are Cheated. New York: Henry Holt and Company, 1947.

Flesch, Rudolph. Why Johnny Can't Read. New York: Harper & Brothers, 1955.

Hechinger, F. M. The Big Red Schoolhouse. New York: Doubleday and Company, Inc., 1959.

Keats, John. Schools Without Scholars. Boston: Houghton Mifflin Company, 1958.

Lanser, Kermit et al. Second Rate Brains. New York: Doubleday and Company, Inc., 1958.

McLellan, G. S. (ed.). America's Educational Needs. New York: The H. W. Wilson Company, 1958.

Woodring, Paul. Let's Talk Sense About Our Schools. New York: The McGraw Hill Book Company, Inc., 1953.

## 2. Periodicals

"Crisis in Education," Life Magazine. Part I, March 24, 1958, page 25; Part II, March 31, 1958, page 93; Part III, April 7, 1958, page 89.

Griswold, A Whitney et al. "Let's Attack The Problems, Not The Schools," Ladies' Home Journal, 71: 54-57 and 184-187, October, 1954.

Hardy, W. G. "Education in Alberta," Chapters 1-6, The Edmonton Journal, February 1-6, 1954.

Katz, Sidney. "The Crisis in Education," Maclean's Magazine, 66: 7-9, 54-59, March 1, 1953.

Katz, Sidney. "The Row Over The Three Rs," Maclean's Magazine, 66: 20-21, 47-51, March 15, 1953.

Rickover, Hyman G. "The World of the Uneducated," *The Saturday Evening Post*, November 28, 1959, pp. 19, 54, 57, 59.

"The Deeper Problems in Education," (editorial), *Life Magazine*, March 31, 1958, p. 32.

## B. PROFESSIONAL AND TECHNICAL PUBLICATIONS

### 1. *Books*

Cochrane, William G. *Sampling Techniques*. New York: John Wiley and Sons, Inc., 1953.

Dewey, John. *How We Think*. New York: D. C. Heath and Company, 1933.

Henderson, Kenneth B. and Pingry, Robert E. "Problem Solving in Mathematics." Chapter 8 of *The Twenty-first Yearbook of The National Council of Teachers of Mathematics*. Washington, D.C.: The National Council of Teachers of Mathematics, 1953.

Kenney, J. F. and Keeping, E. S. *Mathematics of Statistics*. Toronto: D. Van Nostrand Company, Inc., 1954.

Lindquist, E. F. *Statistical Analysis in Educational Research*. New York: Houghton Mifflin Company, 1940.

Moroney, M. J. *Facts from Figures*. Harmondsworth, Middlesex: Penguin Books Limited, 1951.

Parten, Mildred. *Surveys, Polls and Samples*. New York: Harper & Brothers, 1950.

Partlow, Hugh R. *Arithmetic and Reading, Yesterday and Today*: A Comparison of St. Catherines Public School Standards, 1933-38 and 1952-54. Toronto: The Copp Clark Company, Limited, 1955.

Vinacke, W. Edgar. *The Psychology of Thinking*. New York: The McGraw Hill Book Company, Inc., 1952.

Walker, Helen M. and Lev, Joseph. *Statistical Inference*. New York: Henry Holt & Co., 1953.

Wertheimer, Max. *Productive Thinking*. New York: Harper & Brothers, 1945.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Whitehead, Alfred North. The Aims of Education. New York: The Macmillan Company, 1929, and Mentor Books, The New American Library, 1949 (paper-back edition).

Yates, Frank. Sampling Methods for Censuses and Surveys. London: Charles Griffin and Company, Limited, 1949.

2. Periodicals

Boss, Mabel E. "Arithmetic Then and Now," School and Society, 51: 62-64, January 13, 1940.

Buswell, Guy T. "Patterns of Thinking in Solving Problems," University of California Publications in Education, Vol. 12, No. 2, pp. 63-148. Berkely: University of California Press, 1956.

Edmonson, J. B. "The Old Versus The New In Education," The Educational Courier, 22: 18, 19, 46, April, 1952.

Fehr, Howard E. "The Rôle of Insight in the Learning of Mathematics," The Mathematics Teacher, October, 1954, pp. 386-392.

Georges, J. S. "Learning to Solve Problems Intelligently," School Science and Mathematics, December, 1956, pp. 701-707.

MacIntosh, W. A. "The Perennial Crisis in Education," The Canadian School Journal, 31: 208, 209, 215, 217, 256, August-September, 1952.

Sansom, Clarence E. "Sixth Statistical Report, Grade X Survey Tests," The A.T.A. Magazine, 31: 28-33, June, 1950.

Toogood, John. "Our Curriculum Whither Bound?" The Educational Courier, 24: 10, 11, 46-48, April, 1954.

"What Is Right In Our Schools?" (editorial). The Modern Instructor, 22: 513, 514, May, 1954.

## C. REPORTS AND GOVERNMENT PUBLICATIONS

Cameron, Donald A. *et al.* *The Report of the Royal Commission on Education in Alberta.* Edmonton: L. S. Wall, Printer to The Queen's Most Excellent Majesty, 1959.

Hope, J. A. *et al.* *The Report of the Royal Commission on Education in Ontario.* Toronto: Baptist Johnston, Printer to the King's Most Excellent Majesty, 1950.

MacFarlane, R. O. *et al.* *The Report of the Manitoba Royal Commission on Education.* Winnipeg: Queen's Printer, 1959.

Glennon, Vincent J. and Runnicut, G. W. *What Does Research Say About Arithmetic?* Washington, D.C.: Association for Supervision and Curriculum Development, 1952.

Wrightstone, J. Wayne. *Evaluating Achievement in Basic Skills in Newer Versus Conventional Schools*: Summary of Major Research Studies. A report to the Childhood Education Committee of the Governor's Fact-Finding Commission on Education, 1951. Hartford, Connecticut: State of Connecticut, 1951.

*Junior High School Curriculum Guide for Mathematics.* Province of Alberta, Department of Education, 1952.

*Programme of Studies for The Intermediate School and Departmental Regulations Relating to the Grade IX Examination.* Edmonton: A. Shnitka, King's Printer, 1935.

## D. UNPUBLISHED MATERIALS

Lanton, Wandell C. *Comparison of Reading, Arithmetic and Spelling Achievement of Third and Fifth Grade Pupils in 1953 and 1954* (Doctor's Thesis). Evanston, Illinois: Northwestern University, 1954.

*The Three Rs Hold Their Own At The Midcentury.* A summary of research studies compiled by the Research Division of The National Education Association of the United States. Washington, D. C., 1951.

# APPENDICES

## APPENDIX A

Table from the article by Mabel E. Boss, "Arithmetic Then and Now," School and Society, March 23, 1940, p. 391.

### TABLE I

### MEDIAN SCORES IN ARITHMETIC FOR THE FOURTH QUARTERS OF GRADES III, VI AND VIII IN 1916 AND 1938

| Section of Test | III-4 | | VI-4 | | VIII-4 | |
|---|---|---|---|---|---|---|
| | 1916 | 1938 | 1916 | 1938 | 1916 | 1938 |
| A | 18.3 | 13.6 | 26.4 | 23.0 | 32.2 | 28.5 |
| B | 12.2 | 8.7 | 20.6 | 21.6 | 28.3 | 28.8 |
| C | 10.5 | 1.6 | 18.3 | 17.0 | 21.9 | 18.0 |
| D | 12.2 | 0.9 | 20.5 | 20.1 | 25.7 | 19.6 |
| E | 4.8 | 3.4 | 7.1 | 6.1 | 8.4 | 6.8 |
| F | 3.5 | 2.5 | 8.3 | 6.6 | 11.3 | 9.6 |
| G | 3.5 | 0.9 | 6.2 | 5.8 | 7.8 | 7.3 |
| H | 3.8 | 0.0 | 8.1 | 4.0 | 12.0 | 4.2 |
| I | 1.4 | 0.6 | 4.1 | 2.5 | 5.8 | 4.2 |
| J | 2.9 | 1.0 | 5.1 | 3.9 | 5.8 | 4.9 |
| K | 0.0 | 0.0 | 7.4 | 8.9 | 11.7 | 9.5 |
| L | 0.0 | 0.0 | 4.1 | 4.2 | 5.3 | 5.1 |
| M | 2.1 | 0.8 | 4.4 | 4.4 | 5.3 | 5.2 |
| N | 0.0 | 0.0 | 1.8 | 2.0 | 2.7 | 2.7 |
| O | 0.0 | 0.5 | 4.1 | 5.2 | 6.6 | 3.9 |

Scores for all grades show the same trends.

## APPENDIX B

## LETTER REQUESTING THE CO-OPERATION OF THE PRINCIPALS OF THE SELECTED SCHOOLS

Ross Sheppard Composite High School,
13546 - 111 Avenue,
Edmonton, Alberta,

April 28, 1959.

Dear Mr. ----------:

I am writing this letter to request your co-operation in a research project which I have undertaken for the preparation of a master's thesis. I trust you will feel, after having read the following outline, that the routine of your school would be affected very little by the small amount of time required.

The purpose of my investigation is to compare the present Alberta high school entrance standard in mathematics with that of the year 1938. The study will be similar to that carried out (on a larger scale) in the fields of arithmetic and reading by H. R. Partlow in St. Catherines, Ontario; Partlow's study resulted in a doctoral dissertation which has been published (Hugh R. Partlow, *Arithmetic and Reading, Yesterday and Today*. Toronto: The Copp Clarke Company, Ltd., 1955). I plan to administer the 1938 Grade IX Departmental Examination in mathematics (algebra section only) to a sample of the Edmonton Grade IX pupils of this year, and to compare their scores with those of a random sample of the Edmonton Grade IX pupils of 1938. This comparison and its analysis and interpretation will form the core of the study. I plan also to administer the 1938 General Test to the same pupils in order to compare their general ability with that of the 1938 sample, insofar as this test will indicate it. The reason I am not using the Geometry section of the examination is that the course contents of the respective years have some rather considerable differences.

I expect to test a sample of from 500 to 600 pupils. I wish the sample to be as representative as possible of the 1959 Edmonton Grade IX population. Unfortunately the sampling procedures which would be theoretically most desirable from a statistical point of view are the least practicable from the administrative standpoint. After a good deal of discussion with my committee of professors I have decided (subject to the consent of the teachers and administrators concerned) to test all of the Grade IX pupils in the following schools: Eastwood, King Edward Park, King Edward, McKernan, McKay Avenue and Westmount.

The Algebra test will require one and a half hours, and the General Test 45 minutes. Thus, two and a quarter hours of the time of the Grade IX classrooms in these schools will be required, but this need not all be in one block or even in one day -- the two tests could be given on different days.

The testing will require supervision by some staff members, and the relinquishing by them of a class period or so in their own subjects. I realize this is quite a bit to ask at the end of the year when every hour of review is important, but I trust that it will be possible.

The tests should be given as late in the term as possible, having consideration for the requirements of term-end review work which the teachers will want to carry out. I have in mind some time about the 11th or 12th of June -- even later if possible, but earlier if necessary. The principal and staff may choose their own time for holding the tests. It will not be necessary to hold them both on the same day.

The test papers will be delivered to the school and the answer papers picked up. Supervision instructions will be supplied. Scores and comparative data for the participating classes will be available to principals if desired.

Mr. M. J. V. Downey, Director of Personnel and Research for the Edmonton Public School system, has given his consent to my carrying out this project insofar as it will involve the schools. However, the co-operation of principals and teachers will be on an entirely voluntary basis. I am requesting your help so that I may complete this investigation and discover whether today's Grade IX students are better prepared or not so well prepared for the mathematics program of the senior high school as were the students of 21 years ago. There have been many allegations during the last few years that standards in the schools have declined. The hundreds of claims which have been made on both sides of this question by educators and lay people alike have been for the most part expressions of opinion only. It is time some facts were introduced into the controversy, and significant facts can be obtained only by systematic and controlled studies. So far as I know, no such studies have yet been made in Alberta. With your co-operation I hope to arrive at some valid conclusions with respect to the area covered by this piece of research.

Will you please give this request your consideration and bring it to the attention of your teachers? I shall be most grateful if you will fill in the attached sheet and return it to me at your earliest convenience in the enclosed stamped envelope.

Yours sincerely,

Norman D. Muir.

# INFORMATION SHEET

for research study by N. D. Muir

1. Name of school ______________________________

2. Are you willing to co-operate in this study, as outlined in the accompanying letter?

__________
(Yes or No)

3. If you have answered Yes, will you please list the Grade IX classrooms in your school by number or letter or other designation by which they are ordinarily identified. Please give the enrollment also, and if you have made an "ability grouping" of your students, indicate for each classroom whether it is a superior, average, or below average group.

| | Room Number or Letter | Enrollment | Indicate ability grouping (if any) in this column |
|---|---|---|---|
| 1. | | | |
| 2. | | | |
| 3. | | | |
| 4. | | | |
| 5. | | | |

4. Do you feel that the approximate testing date I have mentioned (around the 12th of June) would be satisfactory?

__________

5. Would you like a summary of the results of the testing, with particular reference to the classrooms from your own school?

__________

6. Would you please make here a general statement about the degree of instructional departmentalization in the Junior High rooms in your school.

7. Any comments or suggestions you may have will be most welcome -- just jot them on the reverse side of this sheet.

______________________
Principal's signature

# APPENDIX C

## INSTRUCTIONS

## TO THE SUPERVISING TEACHER

---

### GENERAL INSTRUCTIONS

It is hoped that the writing of this examination will be done as closely as possible in accordance with the usual rules governing the writing of Departmental examinations. Some modification is, however, necessary. Supervising teachers are requested to follow the guide below for the sake of ensuring uniformity in the conditions of writing.

1. Many students will ask why these tests are being given, whether all students are writing them, whether the marks they make will affect their final standing in Grade IX, and other similar questions. The students should be told that these tests are being given for research purposes, that all Grade IX students in certain selected schools are writing them, and that the mark they make will not affect their Grade IX standing. It should be made clear, however, that the scores they make will be used in an important way, and that they are to work on this test just as though it were their final examination. This information appears in a special note TO THE STUDENT on the first page of the examination itself.

2. Some students may ask, "Must we write these tests?" The answer is Yes. This testing has been approved by the proper authority of the Edmonton Public School Board and by the Principal of the school, and all Grade IX students in the selected schools must write. (Students absent for legitimate reasons on the day of the test are, of course, excused).

3. With respect to points (1) and (2) above, the aim is to be frank and honest with the students about why they are required to write the tests, but at the same time to try to ensure that their motivation and effort in writing will be as strong as possible.

4. The package of test papers should be opened by the supervising teacher in the presence of the students.

5. The supervising teacher should distribute the papers face downward on the desks. When all papers have been given out, the students should be told to turn them over, to read the NOTE TO THE STUDENT at the top, and to fill in the required information in ink on the upper part of the first page. After they have done this, they should check to see that the paper is complete (there are five pages).

6. The teacher will then tell the students to commence writing, and will take note of the time. The time for the Mathematics test is one and a half hours, and for the General Test, forty-five minutes.

7. Papers should be written in ink.

8. Supervising teachers are asked to exercise reasonably close surveillance over the students, particularly as they will be sitting in adjoining seats. If a student is discovered copying from another, or allowing another to copy from him, or in any way breaking the usual rules governing the writing of examinations, the supervising teacher should, of course, interfere. The student may be allowed to finish the paper, however; but when he hands it in the teacher should write across the front page a notation to the effect that the student was discovered unmistakably cheating and that his score will therefore be invalid.

## SPECIAL INSTRUCTIONS FOR THE ALGEBRA TEST

1. The time for this test is one and a half hours.

2. No student shall hand in his Algebra paper until at least one hour has elapsed.

3. If a student hands in his paper before the expiration of the full time allowed, he shall remain in his seat and spend the remaining time on some other work unless arrangements have been made in the school for such a student to leave the room. Students who leave the room by special arrangement shall not be allowed to re-enter until after the full time for the examination has elapsed.

4. If a student asks any question regarding details of the paper or the manner of answering, the supervising teacher

may reply according to his discretion, but should avoid giving any assistance by way of interpretation or suggestion.

5. Unused copies of the Algebra test may be kept by the school, and should be turned over to the Principal or to one of the mathematics teachers.

## SPECIAL INSTRUCTIONS FOR THE GENERAL TEST

1. The time for this test is 45 minutes.
2. After the students have filled in the information at the top of the first page, the teacher should go over with them the DIRECTIONS and examples preceding the questions. When all students appear to understand how the answer-spaces should be filled in, they should be told to begin, and the time should be noted.
3. When the students have begun writing, the teacher should move about the room and ascertain that all are going about it in the right way. Students are not allowed to ask questions of the teacher regarding the details of the paper.
4. Unused copies of the General Test are not to be kept by the school, but should be returned along with the answer papers.

---

## IN APPRECIATION

I wish to express my appreciation and thanks to all principals and teachers who have given up instruction time to make this testing possible. I trust that this research project will yield information which will be significant and helpful in the evaluation of our mathematics program. Your kind co-operation has been an important factor in the progress of this study.

Norman D. Muir.

## APPENDIX D

## THE ALGEBRA TEST

### SPECIAL TEST IN ALGEBRA - GRADE IX

Edmonton, June, 1959

---

### TO THE STUDENT

THIS IS A SPECIAL TEST IN ALGEBRA WHICH IS BEING GIVEN TO EDMONTON GRADE IX STUDENTS FOR RESEARCH PURPOSES. THE MARK YOU MAKE ON THIS TEST WILL NOT AFFECT YOUR FINAL GRADING FOR THE YEAR, BUT YOUR SCORE WILL BE USED IN AN IMPORTANT WAY, AND YOU ARE TO WORK ON THIS TEST JUST AS THOUGH IT WERE YOUR FINAL EXAMINATION

STUDENT'S NAME ____________________ BOY / GIRL SCHOOL ___________

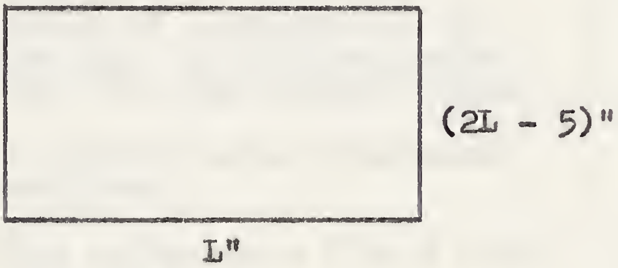
YOUR AGE ON JUNE 30 ____ BIRTHDATE: ______ MONTH ______ DAY ______ YEAR

ARE YOU REPEATING GRADE IX? YES / NO RAW SCORE ___________

(FOR EXAMINER'S USE)

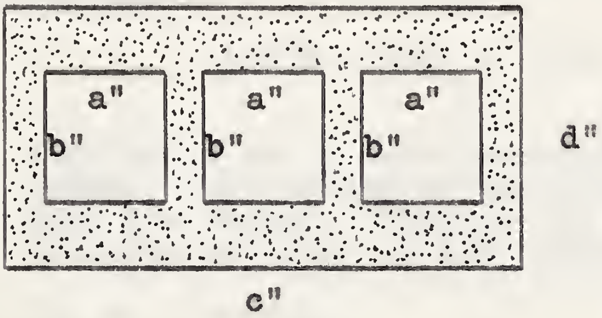
Time: 1½ hours

NOTE: In questions 6, 11, 13, 14, 15, 16, 18 and 20 full solutions are to be shown.

| Values | | |
|---|---|---|
| 1 | 1. Find the value of d eggs at c cents a dozen. | |
| 1 | 2. Reduce to lowest terms: $\frac{15(x + y)}{3y}$ | |
| 1 | 3. How many factors are there in the expression abc(2a + b + c)? | |
| 2 | 4. Write a formula for the perimeter of the rectangle shown. Use p for the perimeter.<br> | |
| 3 | 5. If, in the above rectangle, p = 27, find L. | |
| 2 | 6. If r = 10½, s = 7½, and t = 4½, find the value of<br>$6r - 7s + 8t$ | |

Values

2 7. Write the formula that will give the area of the shaded portion of the following figure:



2 8. How long should it take 2 men to do a job that M men can do in 9 hours?

2 9. The price of a bicycle is increased x%. If the new price is $40, what was the old price?

10. Write the following statements as equations:

1 (a) The number 2n + 9 is 6 less than 4n - 3.

1 (b) Twice the sum of n + 2 and r is 36.

1 (c) 15% of n + 7 is 25 more than 12% of n - 2.

3 11. Remove the parentheses and combine like terms:

$$9x^2 + 3x + 2\left[2x - 5x(x - 1) - 10\right] - (2x^2 + 7x)$$

2 12. From $(3x^2 - 7x + 9)$ subtract $(-5x^2 + 10x - 2)$

Values

3 13. Find the product of $4a^2 - 5a + 3$ and $5 - 2a$

4 14. Solve the following equation and check the solution:

$$3/5\ N - 2.4 = 7.2$$

15. A cylinder has a base containing 5 square inches. The volume of the cylinder depends upon its height. Express the volume of the cylinder in terms of its height, h:

2 (a) by a formula

5 (b) by a graph. (Draw a graph that will show the volume of this cylinder for any height less than 10 inches).

Values

6 16. Solve the following equation and verify the solution obtained:

$$\frac{x - 2}{4} - \frac{x - 4}{6} = \frac{2}{3}$$

17. For each of the following problems write the equation which may be used in its solution. (Write the equations but do not solve them).

4 (a) Two boys have together 250 marbles. One boy has 20 more than twice as many as the other. How many marbles has each boy?

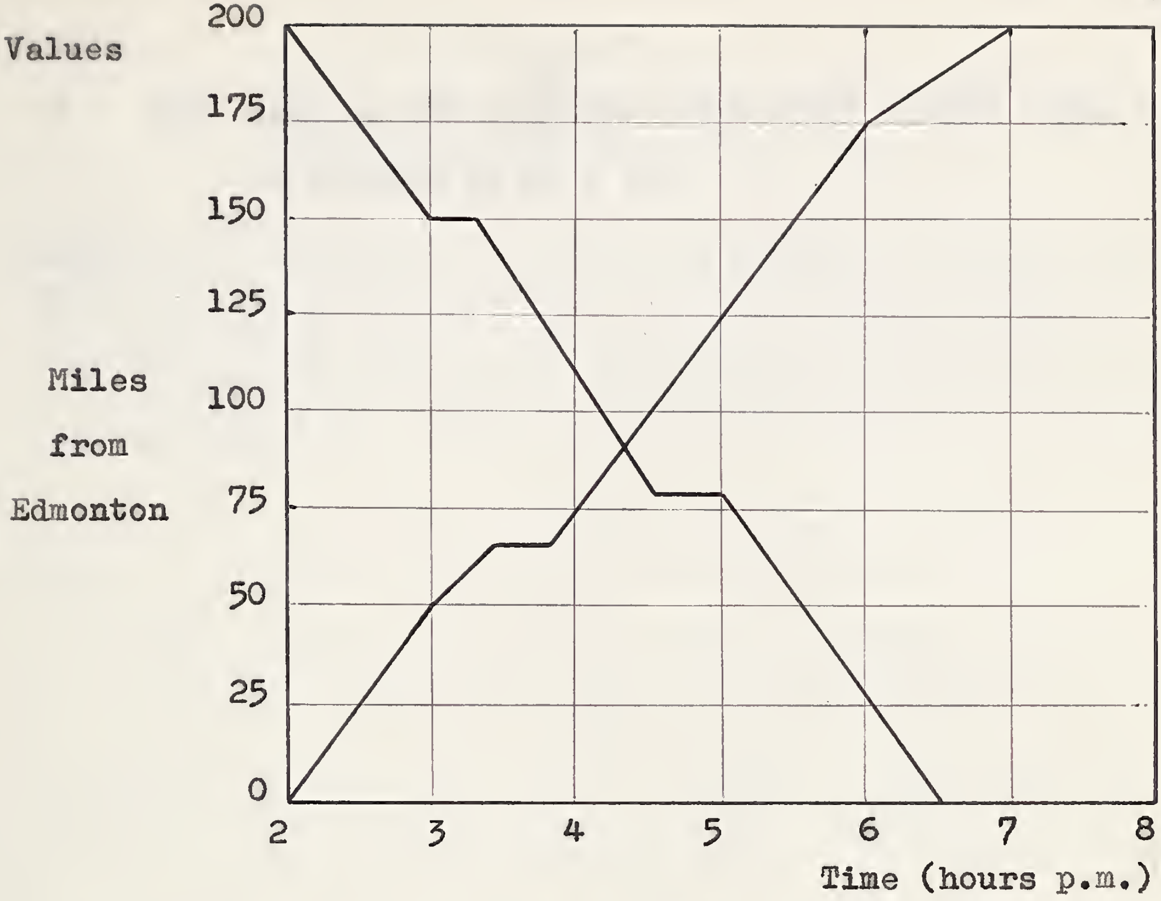
Equation:

4 (b) The width of a field is 40 rods less than the length. The perimeter of the field is 300 rods. Find its width.

Equation:

7 18. Three men compare their taxes. The first finds that he pays $20 more per year than the second and $30 more per year than the third. If the sum of their taxes is $550, how much does each pay?

19. Calgary and Edmonton are 200 miles apart. Car A leaves Calgary for Edmonton at the same instant that Car B leaves Edmonton for Calgary. The accompanying graph pictures the journey made by each car. From the graph answer the following questions:



GRAPH SHOWING JOURNEYS MADE BY CARS A AND B ON HIGHWAY NO. 2, SEPT. 30, 1937

1 (a) At what time does Car A arrive at Edmonton?

2 (b) What was the average speed of Car A between Calgary and Edmonton?

2 (c) At what speed was Car B travelling when it met car A?

2 (d) How far was Car B from Calgary when it stopped for several minutes?

3 (e) At what time did the cars meet?

3 (f) How far from Calgary were the cars when they met?

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Values

3 20. What is the remainder when $20a^3 - 22a^2 + 33a - 18$ is divided by $5a - 3$?

———

75

# APPENDIX E

## SCORING KEY FOR THE ALGEBRA TEST

NOTE: The following key shows a solution for each of the problems on the test. However, a student's solution did not have to be identical to that of the key in order to earn the marks. A key cannot show or anticipate all of the variations which occur among students' solutions, but it is a guide for making as fair and uniform an evaluation as possible.

| Values | | | |
|---|---|---|---|
| 1 | 1. | | $\frac{cd}{12}$ cents |
| 1 | 2. | | $\frac{5(x + y)}{y}$ |
| 1 | 3. | | 4 |
| 2 | 4. | (1 mark for correct formula unsimplified) | $p = 6L - 10$ |
| 3 | 5. | | $L = 6\ 1/6$ |
| 2 | 6. | (1 mark for correct substitution). | 46½ |
| 2 | 7. | (1 mark if the right hand side only is given) | $A = cd - 3ab$ |
| 2 | 8. | | $\frac{9M}{2}$ hours |

Values

2 9. $\$\ \dfrac{4000}{100 + x}$

1 10. (a) $2n + 9 = 4n - 9$

(or equivalent)

1 (b) $2(n + 2 + r) = 36$

(or equivalent)

1 (c) $.15(n + 7) = .12(n - 2) + 25$

3 11. $9x^2 + 3x + 2 \quad 2x - 5x^2 + 5x - 10 \quad - 2x^2 - 7x$

$= 9x^2 + 3x + 4x - 10x^2 + 10x - 20 - 2x^2 - 7x$

$= -3x^2 + 10x - 20$

(1 mark for each step)

2 12.

$$\begin{array}{r} 3x^2 - 7x + 9 \\ -5x^2 + 10x - 2 \\ \hline \end{array}$$

Subtracting, $8x^2 - 17x + 11$

(Deduct one mark for each error in the answer)

3 13. $(4a^2 - 5a + 3)(5 - 2a)$

$= 20a^2 - 25a + 15 - 8a^3 + 10a^2 - 6a$

$= -8a^3 + 30a^2 - 31a + 15$

( 1 mark for each partial product, 1 for the sum)

Values

4 14. Solve: $3/5 N - 2.4 = 7.2$

Multiply by 10: $6N - 24 = 72$ (1 mark)

Add 24: $6N = 96$ (1 mark)

Divide by 6: $N = 16$ (1 mark)

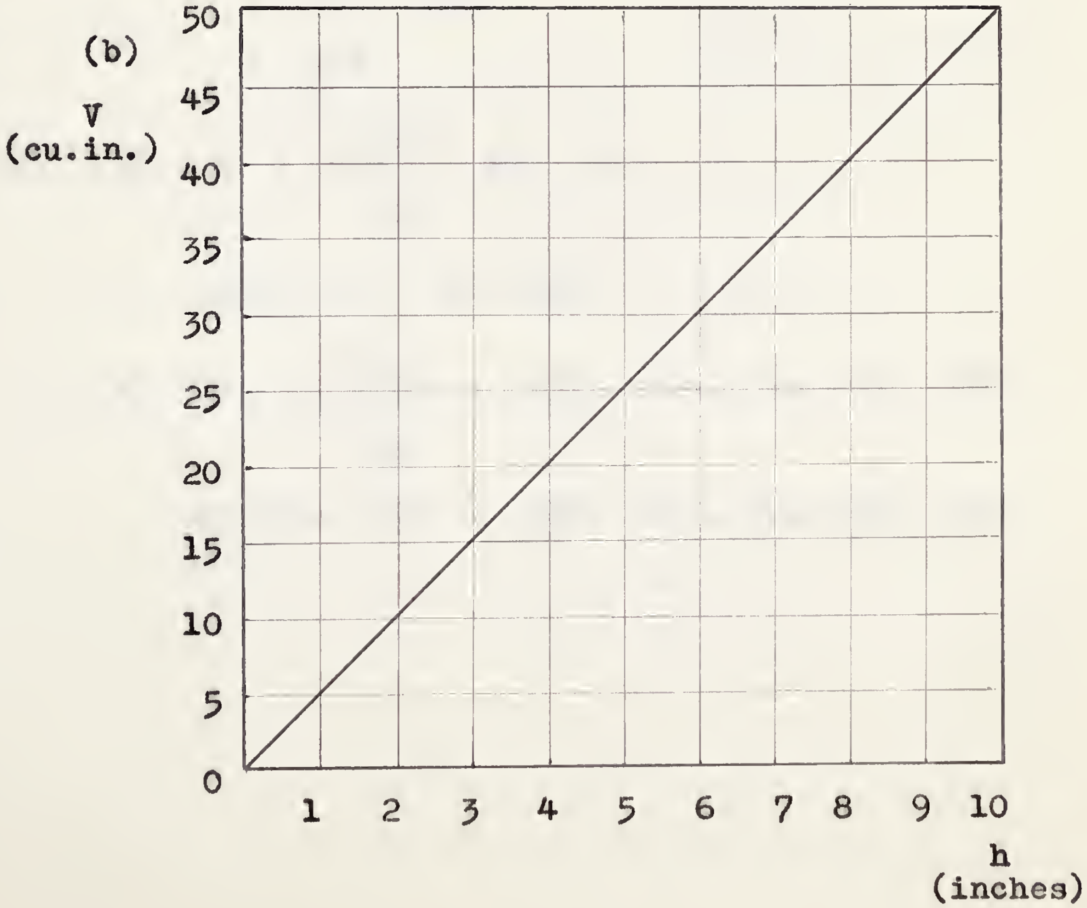
Check:

(1 mark)

| L.H.S. | R.H.S. |
|---|---|
| $\frac{3}{5}(16) - 2.4$ | 7.2 |
| $48/5 - 2.4$ | |
| $9.6 - 2.4$ | |
| 7.2 | |

2 15. (a) $V = 5h$

5 (b)



Values Marks for the graph:

1 for labelling the axes
1 for appropriate scales
3 for the graph itself, complete and correct

6 16. Solve: $\frac{x-2}{4} - \frac{x-4}{6} = \frac{2}{3}$

Multiply by 12: $3(x-2) - 2(x-4) = 8$ (2 marks)

$3x - 6 - 2x + 8 = 8$ (1 mark)

$x = 6$ (1 mark)

Verification (2 marks):

| L.H.S. | R.H.S. |
|---|---|
| $\frac{6-2}{4} - \frac{6-4}{6}$ | $\frac{2}{3}$ |
| $= 4/4 - 2/6$ | |
| $= 2/3$ | |

4 17. (a) $x = 2(250 - x) + 20$

OR

$250 - x = 2x + 20$

4 (b) $2x + 2x + 80 = 300$; i.e., $4x + 80 = 300$

OR

$2x + 2x - 80 = 300$; i.e., $4x - 80 = 300$

Values

7 18. Solution:

Let A's tax be \$x )
Let B's tax be \$(x - 20) ) (2 marks)
Let C's tax be \$(x - 30) )

Then $x + x - 20 + x - 30 = 550$ (2 marks)

$$3x - 50 = 550$$

$$3x = 600$$

$$x = 200$$ (2 marks)

Then A pays \$200, B pays \$180, )
C pays \$170. ) ( 1 mark)

| | | | | |
|---|---|---|---|---|
| 1 | 19. | (a) | | 6:30 p.m. |
| 2 | | (b) | (1 mark for 44 m.p.h. or 45 m.p.h.) | 44 4/9 m.p.h. |
| 2 | | (c) | | 50 m.p.h. |
| 2 | | (d) | (1 mark for 130 miles or 140 miles) | 135 miles |
| 3 | | (e) | (2 marks for 4:15 p.m. or 4:25 p.m., 1 for 4:30 p.m.) | 4:20 p.m. |
| 3 | | (f) | (1 mark for 105 miles or 115 miles) | 110 miles |

3 20. $2a - 3$

75

# APPENDIX F

## GENERAL TEST FOR GRADE IX

## EDMONTON, JUNE, 1959

---

### TO THE STUDENT

THIS TEST IS BEING GIVEN TO EDMONTON GRADE IX STUDENTS FOR RESEARCH PURPOSES. THERE IS NO PASSING OR FAILING MARK; IT IS A TEST OF YOUR GENERAL KNOWLEDGE AND ABILITY. YOU SHOULD ANSWER ALL OF THE QUESTIONS AS WELL AS YOU CAN.

STUDENT'S NAME ____________________ BOY / GIRL SCHOOL ____________

YOUR AGE AT JUNE 30 _____ BIRTHDATE: ____ MONTH ____ DAY ____ YEAR

ARE YOU REPEATING GRADE IX? YES / NO RAW SCORE ________

(For Examiner's Use)

Time: 45 minutes.

Directions:

Find the correct answer to each of the following questions and put the answers or their corresponding numbers in the brackets at the right.

For example: Which one of the following five things is round? (1) a rake; (2) an axe; (3) a ball; (4) a house; (5) a book. ( 3 )

The cost of 7 pencils at 4 cents each is (?) cents. ( 28¢ )

Hand is to glove and head is to cap as foot is to: (1) mitten; (2) stocking; (3) toque; (4) collar; (5) belt. ( 2 )

ANSWER THE FOLLOWING QUESTIONS IN THE SAME MANNER. DO NOT SPEND TOO MUCH TIME ON ANY ONE ITEM.

1. The opposite of closed is (1) tied; (2) open; (3) fast; (4) loose; (5) close ( )

2. A word meaning the opposite of strange is: (1) familiar; (2) funny; (3) mysterious; (4) amusing; (5) foreign. ( )

3. One number is wrong in the following series. What should that number be?

   2 5 7 5 2 5 7 5 2 6 ( )

4. The two words necessary and essential mean (1) the same; (2) the opposite; (3) neither the same nor the opposite. ( )

5. How many yards are there in 16½ ft.? ( )

6. A chimney is most like (1) a damper; (2) a flue; (3) a cowl; (4) a furnace; (5) an umbrella. ( )

7. A teepee is a kind of (1) fly; (2) skin; (3) tent; (4) snare; (5) boat. ( )

8. What is the cost of 12 cakes at 3 for 5 cents? ( )

9. To conquer means (1) to succumb; (2) to defy; (3) to torment; (4) to overcome; (5) to vanish. ( )

10. How many times must you add 2½ to 7 to get 22? ( )

11. A fragrant flower is always (1) beautiful; (2) colored; (3) sweet-smelling; (5) sun-drenched. ( )

12. What is said of a person constitutes his (1) character; (2) reputation; (3) disposition; (4) temperament; (5) feelings. ( )

13. If 6 and 5 make 11, write C in the brackets unless 8 and 2 also make 11 in which case write D instead. ( )

14. An officious person is generally (1) industrious; (2) useful; (3) likeable; (4) meddlesome; (5) helpful. ( )

15. Ducks are to paddle as fish are to (1) quack; (2) float; (3) breathe; (4) eat; (5) swim. ( )

16. Cat is to tiger as monkey is to (1) baboon; (2) elephant; (3) peanut; (4) cage; (5) jungle ( )

17. A farmer has fewer sheep than cattle and more hogs than cattle. Has he more or fewer hogs than sheep? ( )

18. How many pieces of ribbon each 4 inches long can be cut from an eight yard piece of ribbon? ( )

19. Which word in each group given below means something most unlike the other three in the group? Put its number in the bracket at the right.

For example: (1) red (2) fat (3) blue (4) yellow ( 2 )

(a) (1)pie (2)cake (3)broom (4) bread ( )

(b) (1)play (2)sorrow (3)fun (4)merriment ( )

(c) (1)break (2)crush (3)smash (4)scatter ( )

(d) (1)follow (2)fetch (3)bring (4)carry ( )

(e) (1)green (2)crimson (3)red (4)pink ( )

(f) (1)mule (2)dog (3)fish (d)zebra ( )

(g) (1)blunder (2)error (3)mistake (4)plan ( )

(h) (1)grieve (2)envy (3)lament (4)wail ( )

(i) (1)infant (2)child (3)mother (4)babe ( )

(j) (1)protect (2)escape (3)guard (4)defend ( )

20. When Sam's age is added to George's, the total is 24 years. What will be the total of their ages 8 years from now? ( )

21. There are 8 boys in the class and 4/5 of the class are girls. How many girls are there in the class? ( )

22. I judged by the scowl on his face that he must be (1) injured; (2) frightened; (3) angry; (4) pleased; (5)sorry. ( )

23. With all his wealth he was unable to purchase good (1) clothes; (2) health; (3) houses; (4) dinners; (5) cigars. ( )

24. On the lines below, each number is derived in a certain way from the number coming before it. Find in what way this is done in each line and then write in the brakcets the two numbers that should come next.

4 8 12 16 20 ( )( )

64 32 16 8 4 ( )( )

39 35 31 27 23 ( )( )

6 11 21 26 36 ( )( )

1 3 6 8 16 ( )( )

25. 3% of 2,000 is 6% of ( )

26. A wheel always has (1)an axle; (2) a rubber tire; (3) spokes; (4) a circumference; (5) a hub cap. ( )

27. A watch which loses 20 seconds each day is set at correct time on Monday at noon. What time is it by the watch when it is 12 noon on Friday by true time? ( )

28. The goods not destroyed by the fire were damaged by the (1) firemen; (2) water; (3) onlookers; (4) policemen; (5) darkness. ( )

29. Preparing for a hard day's climb he put on his (1) slippers; (2) spectacles; (3) vest; (4) raincoat; (5) heavy boots. (   )

30. Bob is taller than Jack. Dick is taller than Bob. Who is the shortest? (   )

31. My brother is 4 years less than 3 times as old as I am. I am 8 years old. How old is my brother? (   )

32. Knowledge of the truth saves one from being (1) deceived; (2) pardoned; (3) discouraged; (4) unlucky; (5) optimistic. (   )

33. A book marked $5 is sold for $1. What is the per cent by which the price was reduced? (   )

34. In a race, D was ahead of B. C was ahead of A. D and A were just even. Which won, A, B, C or D? (   )

35. If my aunt is Jane's mother, what relation is Jane to my father? (1) daughter; (2) sister; (3) niece; (4) cousin. (   )

36. Last year I owned a 2/3 share in a mine. Since then I sold 2/5 of my share. What part of the mine do I own now? (   )

37. If an automobile goes 150 yards in 10 seconds, how many feet does it go in 1/3 second? (   )

38. John is older than Fred. Mary is younger than John. Mary is older than Fred. If the first two statements are true, the last is (1) true; (2) false; (3) not certain. (   )

39. Three girls are sitting in a row. Eva is to the right of Edith. Edith is to the right of Marie. Which girl is in the middle? (   )

40. Cap is to head as stocking is to (1) wrist; (2) eyes; (3) wear; (4) foot; (5) silk. (   )

41. How many minutes are there from 4:50 p.m. to 6:20 p.m. of the same day? (   )

42. Cultivate is to garden as manicure is to (1) flowers; (2) teeth; (3) nails; (4) vegetables; (5) hoe. (    )

43. Honesty is to virtue as treachery is to (1) enemy; (2) vice; (3) faithfulness; (4) deceit; (5) fair play. (    )

44. What fraction is midway between ½ and 1¼? (    )

45. 3½ miles an hour is how far in 3½ hours? (    )

46. A cubical box of wood is 5 inches long on each edge. What is the area of its total surface? (    )

47. Contempt is to respect as repulsion is to (1) attraction; (2) cowardice; (3) courage; (4) distraction; (5) distaste. (    )

48. Organ is to music as pencil is to (1) lead; (2) piano; (3) paper; (4) eraser; (5) sketch (    )

49. If 2 girls can make 4 dresses in 2 weeks, how long should it take one girl to make one dress? (    )

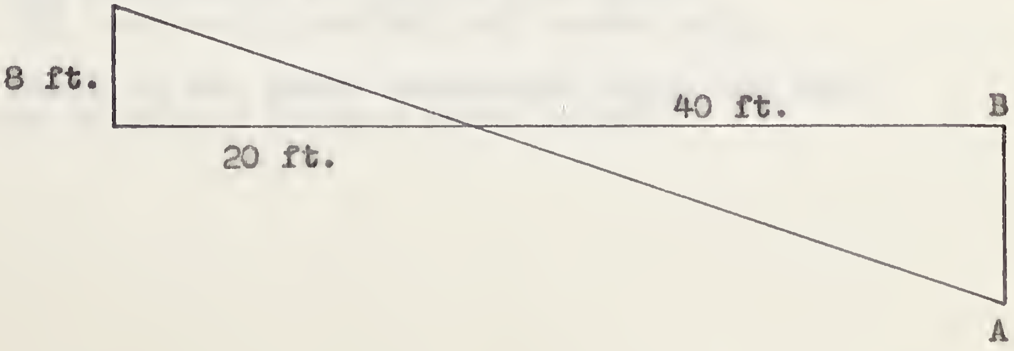
50. Coax is to persuade as try is to (1) beg; (2) succeed; (3) fail; (4) threated; (5) endure. (    )

51. Riches are to miser as poverty is to (1) spendthrift; (2) gentleman; (3) plumber; (4) recluse; (5) peasant. (    )

52. The statement that a dead rabbit thinks is (1) wicked; (2) unjust; (3) misleading; (4) absurd; (5) believable. (    )

53. 3 is related to 9 and 5 is related to 25 as 7 is related to (    )

54. How long is AB in the drawing below? (    )



55. What number contains 9 as many times as ¾ contains 1/8? ( )

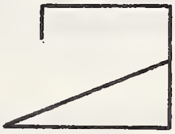
56. If the following words were arranged in order from largest to smallest measure, with what letter would the middle word begin?--gallon, bushel, quart, peck, pint. ( )

57. Write the letter that precedes the third letter after N in the alphabet. ( )

58. If 3½ tons of hay cost $28, what will 5½ tons cost? ( )

59. In a foreign language, some rice = Ckep Fuhl
some flour = Drig Fuhl
some rice and sone flour = Ckep Brag Drig Fuhl
The word that means "and" begins with what letter? ( )

60. Draw in the bracket on the right the following figure as it would appear if seen in a mirror. [ ]



61. If the words below were arranged to make a good sentence, with what letter would the final word begin?

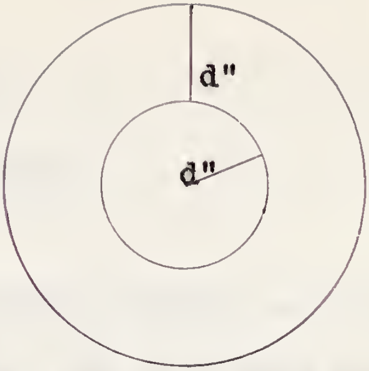
to good best tried boy his a Freddie be ( )

62. Which of the six statements below explains this proverb: "Rome was not built in a day"?

(1) Carpenters usually work in the daytime.
(2) God helps those who help themselves.
(3) Great achievements require time and effort.
(4) Vain people often come to grief.
(5) Prevention is better than cure.
(6) Rome was a wealthy and wicked city. ( )

Which of the above statements means the same as "A stitch in time saves nine"? ( )

63. From a circular sheet of metal a circular hole is punched at the centre. The dimensions are shown in the figure. If the area of the hole is 20 sq. in., the area of the remaining ring of metal is ( )



64. The accident was probably due to his (1) caution; (2) perseverence; (3) foresight; (4) rashness; (5) courage. ( )

65. A large branch of a tree has five smaller branches, and each smaller branch has four tiny branches. How many branches are there altogether? ( )

66. On January 1, 1936, a boy was twice as old as he was on Jan. 1, 1931. How old will he be on January 1, 1940? ( )

67. The opposite of hostile is (1) vicious; (2) tender; (3) generous; (4) friendly; (5) optimistic. ( )

68. Which letter in the word "goldfinch" has the same place in the word as it has in the alphabet? ( )

69. A wire 18 inches long is cut into two pieces. One piece is 4/5 as long as the other. How long is the longer piece? ( )

70. Which of the following words would come last in the dictionary? (1) brutal; (2) blithe; (3) bury; (4) brought; (5) butter. ( )

# APPENDIX G

## STATISTICAL THEORY

This appendix gives additional details of the statistical tests used in Chapter VI. The following points are discussed somewhat more fully than in the text of the thesis: (1) confidence limits for the difference of the population means, (2) the F-test, (3) formulae for finding the standard deviation of a sample. In addition, a method is given for finding confidence limits for the variance of a population from sample statistics.

### 1. Confidence Limits for the Difference of the Population Means

The following quotation from Mathematics of Statistics by Kenney and Keeping, while not a complete derivation, provides further background for formula (2), which is given on page 75 of the text:

> If we have two samples of sizes $N_1$ and $N_2$, means $\bar{x}_1$ and $\bar{x}_2$, and standard deviations $s_1$ and $s_2$, we can form confidence limits for the difference of the means $\mu_1$ and $\mu_2$ in the two populations from which the samples are supposed to be taken, on the hypothesis that these populations have the same variance. If the confidence limits include the value zero, we can say that the

means of the two populations are not different, or, in other words, the hypothesis that the two samples come from populations with the same mean is not rejected at the chosen level.

If we denote the degrees of freedom for the two samples by $n_1$ and $n_2$, we can prove that the quantity

$$t = \left[\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)\right] \Big/ \hat{\sigma}_{12} \qquad (2)$$

where $\hat{\sigma}_{12}$ is an unbiased estimate of the variance of the numerator, is distributed as Student's _t_, with $n_1 + n_2$ degrees of freedom.

On the hypothesis of a common $\sigma^2$, the variance of $\bar{x}_1$ is $\sigma^2/N_1$ and that of $\bar{x}_2$ is $\sigma^2/N_2$, so that the variance of $\bar{x}_1 - \bar{x}_2$ (the samples being independent) is

$$\sigma^2 \left[\frac{1}{N_1} + \frac{1}{N_2}\right] = \sigma^2(N_1 + N_2)/N_1N_2 \qquad (3)$$

The estimate of $\sigma^2$ is taken as a weighted mean of estimates from the two samples separately, weighted according to their respective degrees of freedom. The estimate from the first sample is $N_1s_1^2/n_1$ and that from the second sample is $N_2s_2^2/n_2$, so that

$$\hat{\sigma}^2 = (N_1s_1^2 + N_2s_2^2)/(n_1 + n_2)$$

$$= \frac{N_1s_1^2 + N_2s_2^2}{n_1 + n_2} \cdot \frac{N_1 + N_2}{N_1N_2} \qquad (4)$$

With this value of $\hat{\sigma}_{12}^2$, t (formula (2)) has the Student t-distribution, and confidence limits can be found by putting t equal to the appropriate value corresponding to $n_1 + n_2$ degrees of freedom.[1]

For large samples, $n_1$ and $n_2$ may be taken as $N_1$ and $N_2$ respectively.

## 2. The F-test

The test which is outlined above assumes that the populations from which the two samples are drawn have the same variance. The F-test is a check on whether this assumption is justified. The ratio of the population variances is estimated from sample statistics and the estimate is referred to a table of the F-distribution (which takes into account the sizes of the samples) for a determination of its significance.

If there is no reason for believing that the variances of the two populations are different, the null hypothesis that $\sigma_1^2/\sigma_2^2 = 1$ is assumed and the probabilities read from the F-table are doubled. The estimates of $\sigma_1^2$ and $\sigma_2^2$ are respectively $N_1 s_1^2/n_1$ and $N_2 s_2^2/n_2$, so that

$$F = \frac{N_1 s_1^2}{n_1} \bigg/ \frac{N_2 s_2^2}{n_2} \qquad (5)$$

---

[1] Op. cit, p. 184-5. See footnote 41, p. 73.

The distribution of F is independent of the assumed common variance $\sigma^2$, and depends only on the numbers $n_1 = N_1 - 1$ and $n_2 = N_2 - 1$. For samples as large as those used in this study, $n_1$ and $n_2$ are taken as equal to $N_1$ and $N_2$ respectively, so that F is simply the ratio of the sample variances.

## 3. Formulae for Finding the Standard Deviation of a Sample.

The standard deviation of a distribution is defined as the root-mean-square average of the deviations of the observed values from their mean. The formula

$$s_x = \left[ \frac{1}{N} \sum_i f_i (x_i - \bar{x})^2 \right]^{\frac{1}{2}}, \text{ where } N = \sum_i f_i \qquad (6)$$

is simply a re-statement of the definition. In formula (6), the $x_i$ are the observed values of the distribution, the $f_i$ are their respective frequencies, and $\bar{x}$ is their mean.

The formula may be re-arranged thus:

$$\begin{aligned} N s_x^2 &= \sum f_i (x_i - \bar{x})^2 \\ &= \sum f_i (x_i^2 - 2x_i\bar{x} + \bar{x}^2) \\ &= \sum f_i x_i^2 - 2\bar{x} \sum f_i x_i + \bar{x}^2 \sum f_i \qquad (7) \end{aligned}$$

But $\sum f_i x_i = N\bar{x}$ and $\sum f_i = N$. Substituting these, we have

$$Ns_x^2 = \sum f_i x_i^2 - 2N\bar{x}^2 + N\bar{x}^2$$

$$= \sum f_i x_i^2 - N\bar{x}^2$$

$$= \sum f_i x_i^2 - \frac{1}{N}(\sum f_i x_i)^2 \qquad (8)$$

This is the usual form for the calculation of $s_x$ for a set of ungrouped values of x. However, if the sample is large, the values may be grouped, and a "coded" variate u introduced to simplify the arithmetic. A suitable fixed value $x_o$ (= the class mark for one of the central classes of the distribution) is subtracted from each class mark $x_i$; then if c is the class interval, $x_i$ is equal to $cu_i + x_o$. The $u_i$ are consecutive small integers when the classes are all equal in width. Now, $\bar{x} = c\bar{u} + x_o$, so that $x_i = c(u_i - \bar{u})$. Hence,

$$\sum f_i(x_i - \bar{x})^2 = c^2 \sum f_i(u_i - \bar{u})^2$$

$$= c^2 \sum f_i u_i^2 - \frac{1}{N}(\sum f_i u_i)^2$$

$$= c^2 N s_u^2, \qquad (9)$$

where $s_u^2$ is the variance of the u variate. Thus,

$$s_x^2 = c^2 s_u^2 ,$$

and therefore the standard deviation $s_x$ is equal to $cs_u$.

### 4. Confidence Limits for the Population Standard Deviation

Reference was made on page 60 in the text to a method of finding confidence limits for the standard deviation of a population, using sample statistics. The nature of the particular test used in this study is indicated very briefly by the following quotation from *Facts from Figures*, an introduction to statistics by M. J. Moroney:

> The problem of laying down confidence intervals for the standard deviation of the population when we have an estimate based on *large* samples may be (solved by) using the standard error of the standard deviation $\sigma/\sqrt{2N}$ as our standard error of estimate. We may take it that we may be 95 per cent sure that the population standard deviation will not differ from the large sample standard deviation by more than two standard errors.[2]

It should be pointed out that the $\sigma$ in the above quotation refers to the standard deviation of the *sample*, for which the symbol s is used throughout this study.

---

[2] M. J. Moroney, *Facts from Figures* (Harmondsworth, Middlesex: Penguin Books, Limited, 1951), page 245.

# APPENDIX H

## STATISTICAL CALCULATIONS

1. The standard deviation of the 1959 scores on the Algebra Test.

| Class Interval | Class Mark x | Frequency f | u | fu | fu(u) $= fu^2$ | $f(u + 1)^2$ |
|---|---|---|---|---|---|---|
| 0-4 | 2 | 20 | -4 | -80 | 320 | 180 |
| 5-9 | 7 | 61 | -3 | -183 | 549 | 244 |
| 10-14 | 12 | 56 | -2 | -112 | 224 | 56 |
| 15-19 | 17 | 53 | -1 | -53 | 53 | 0 |
| 20-24 | 22 | 46 | 0 | 0 | 0 | 46 |
| 25-29 | 27 | 43 | 1 | 43 | 43 | 172 |
| 30-34 | 32 | 38 | 2 | 76 | 152 | 342 |
| 35-39 | 37 | 48 | 3 | 144 | 432 | 768 |
| 40-44 | 42 | 45 | 4 | 180 | 720 | 1125 |
| 45-49 | 47 | 27 | 5 | 135 | 675 | 972 |
| 50-54 | 52 | 23 | 6 | 138 | 828 | 1127 |
| 55-59 | 57 | 13 | 7 | 91 | 637 | 832 |
| 60-64 | 62 | 4 | 8 | 32 | 256 | 324 |
| 65-69 | 67 | 4 | 9 | 36 | 324 | 400 |
| 70-74 | 72 | 0 | 10 | 0 | 0 | 0 |
|  |  | 481 |  | 447 | 5213 | 6588 |
|  |  |  |  | X2 |  |  |
|  |  |  |  | 894 |  |  |

Charlier check: $\sum f(u + 1)^2 = \sum fu^2 + 2\sum fu + \sum f$

Left hand side $= 6588$

Right hand side $= 5213 + 894 + 481 = 6588$

To find $s_u^2$, the variance of the u variate, formula (8), page 77 in the text, is used, expressed in terms of u:

$Ns_u^2 = \sum f_i u_i^2 - \frac{1}{N}(\sum f_i u_i)^2$. Substituting values from the tabulated calculation above,

$$481 s_u^2 = 5213 - \frac{1}{481}(447)^2$$

$$= 4797.60$$

Then $s_u^2 = 4797.60/481 = 9.974$

$s_u = (9.974)^{\frac{1}{2}} = 3.158$

Then $s_x = cs_u = 5(3.158) = 15.79$; and

$s_x^2 = 249.22$

The standard deviation is therefore 15.79 and the variance is 249.22.

2. The standard deviation of the 1959 scores on the General Test.

| Class Interval | Class Mark $x$ | Frequency $f$ | $u$ | $fu$ | $fu(u) = fu^2$ | $f(u + 1)^2$ |
|---|---|---|---|---|---|---|
| 0-4 | 2 | 0 | -11 | 0 | 0 | 0 |
| 5-9 | 7 | 0 | -10 | 0 | 0 | 0 |
| 10-14 | 12 | 0 | -9 | 0 | 0 | 0 |
| 15-19 | 17 | 0 | -8 | 0 | 0 | 0 |
| 20-24 | 22 | 1 | -7 | -7 | 49 | 36 |
| 25-29 | 27 | 3 | -6 | -18 | 108 | 75 |
| 30-34 | 32 | 3 | -5 | -15 | 75 | 48 |
| 35-39 | 37 | 14 | -4 | -56 | 224 | 126 |
| 40-44 | 42 | 30 | -3 | -90 | 270 | 120 |
| 45-49 | 47 | 26 | -2 | -52 | 104 | 26 |
| 50-54 | 52 | 55 | -1 | -55 | 55 | 0 |
| 55-59 | 57 | 67 | 0 | 0 | 0 | 67 |
| 60-64 | 62 | 71 | 1 | 71 | 71 | 284 |
| 65-69 | 67 | 77 | 2 | 154 | 308 | 693 |
| 70-74 | 72 | 70 | 3 | 210 | 630 | 1120 |
| 75-79 | 77 | 31 | 4 | 124 | 496 | 775 |
| 80-84 | 82 | 17 | 5 | 85 | 425 | 612 |
| 85-89 | 87 | 1 | 6 | 6 | 36 | 49 |
|  |  | 466 |  | 357 | 2851 | 4031 |
|  |  |  |  | X2 |  |  |
|  |  |  |  | 714 |  |  |

Charlier check: $\sum f(u + 1)^2 = \sum fu^2 + 2\sum fu + \sum f$

Left hand side = 4031

Right hand side = 2851 + 714 + 466 = 4031

Using formula (8), in terms of u,

$$Ns_u^2 = \sum f_i u_i^2 - \frac{1}{N}(\sum f_i u_i)^2,$$ and substituting,

$$466 s_u^2 = 2851 - \frac{1}{466}(357)^2$$

$$= 2577.504$$

Then $$s_u^2 = 2577.504/466$$

$$= 5.5311$$

Hence, $$s_u = (5.5311)^{1/2} = 2.352;$$ and therefore

$$s_x = 5(2.352) = 11.76$$

The standard deviation is 11.76.

### 3. The standard deviation of the 1938 scores on the General Test.

| Class Interval | Class Mark x | Frequency f | u | fu | fu(u) = $fu^2$ | $f(u + 1)^2$ |
|---|---|---|---|---|---|---|
| 0-4 | 2 | 0 | -11 | 0 | 0 | 0 |
| 5-9 | 7 | 0 | -10 | 0 | 0 | 0 |
| 10-14 | 12 | 0 | -9 | 0 | 0 | 0 |
| 15-19 | 17 | 0 | -8 | 0 | 0 | 0 |
| 20-24 | 22 | 0 | -7 | 0 | 0 | 0 |
| 25-29 | 27 | 0 | -6 | 0 | 0 | 0 |
| 30-34 | 32 | 1 | -5 | -5 | 25 | 16 |
| 35-39 | 37 | 4 | -4 | -16 | 64 | 36 |
| 40-44 | 42 | 20 | -3 | -60 | 180 | 80 |
| 45-49 | 47 | 29 | -2 | -58 | 116 | 29 |
| 50-54 | 52 | 25 | -1 | -25 | 25 | 0 |
| 55-59 | 57 | 47 | 0 | 0 | 0 | 47 |
| 60-64 | 62 | 35 | 1 | 35 | 35 | 140 |
| 65-69 | 67 | 27 | 2 | 54 | 108 | 243 |
| 70-74 | 72 | 9 | 3 | 27 | 81 | 144 |
| 75-79 | 77 | 6 | 4 | 24 | 96 | 150 |
| 80-84 | 82 | 0 | 5 | 0 | 0 | 0 |
| 85-89 | 87 | 0 | 6 | 0 | 0 | 0 |
| | | 203 | | -24 | 730 | 885 |
| | | | | X2 | | |
| | | | | -48 | | |

Charlier check: $\sum f(u+1)^2 = \sum fu^2 + 2\sum fu + \sum f$

Left hand side = 885

Right hand side = 730 + (-48) + 203 = 885.

Using formula (8) in terms of u,

$$Ns_u^2 = \sum f_i u_i^2 - \frac{1}{N}(\sum f_i u_i)^2, \text{ and substituting,}$$

$$203s_u^2 = 730 - \frac{1}{203}(-48)^2$$

$$= 718.65$$

Then $s_u^2 = 718.65/203$

$$= 3.540$$

Hence, $s_u = (3.540)^{1/2} = 1.882$; and therefore

$$s_x = 5(1.882) = 9.410.$$

The standard deviation is 9.410.

4. Confidence limits for the population mean, General Test, 1938.

Formula (1), page 74 in the text, may be re-arranged as follows:

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{N}} .$$

For the 1938 General Test, $\bar{x} = 56.49$, $s = 9.410$, $N = 203$; and for the 99 per cent confidence limits using a large sample the value of t is 2.576. Hence,

$$\begin{aligned} \mu &= 56.49 \pm \frac{2.576\ (9.410)}{\sqrt{203}} \\ &= 56.49 \pm 1.70 \\ &= 54.79 \text{ or } 58.19 \end{aligned}$$

Accordingly there is a 99 per cent chance that the population mean raw score would be between 54.79 and 58.19, which represents a deviation of less than two marks either way from the sample mean.

### 5. Confidence limits for the population mean, General Test, 1959.

Formula (1), quoted on the preceding page, is again used. For this sample, $\bar{x} = 60.78$, $s = 11.76$, $N = 466$, and the correct value of $t$ is 2.576. Hence,

$$\mu = 60.78 \pm \frac{2.576(11.76)}{\sqrt{466}}$$

$$= 60.78 \pm 1.40$$

$$= 59.38 \text{ or } 62.18.$$

Thus there is a 99 per cent chance that the population mean would lie between 59.38 and 62.18, which represents a deviation of less than one and one-half marks either way from the sample mean.

### 6. Confidence limits for the difference of the mean raw scores on the General Test for the 1938 and 1959 samples.

The required data for this test are as follows:

| | N | $\bar{x}$ | $s_x$ |
|---|---|---|---|
| 1959 Sample (1) | 466 | 60.78 | 11.76 |
| 1938 Sample (2) | 203 | 56.49 | 9.41 |

Before carrying out the calculation for the confidence limits, the significance of the ratio of the estimated population variances is determined by an F-test. The variance of the 1959 sample is $(11.76)^2 = 138.3$, and that of the 1938 sample is $(9.41)^2 = 88.55$. Since the samples are large, these variances are used as the estimates of the variances of the respective populations. Their ratio, $138.3/88.55 = 1.56$. This is an even larger figure than that given in the table of the distribution of F for the 1 per cent level of significance. Consequently the ratio is significant at a level below 2 per cent for a two-tailed test. There is a probability of less than 2 per cent of obtaining by pure chance sample variances as different as these; it may thus be concluded that the variances of the two populations are not different. Confidence limits for the difference of the population means may now be determined.

The estimate of the common variance of the two populations is given by

$$\hat{\sigma}_{12}{}^2 = \frac{N_1 s_1{}^2 + N_2 s_2{}^2}{n_1 + n_2} \cdot \frac{N_1 + N_2}{N_1 N_2}$$

For large samples, $n_1$ and $n_2$ are taken as $N_1$ and $N_2$ respectively. Substituting appropriately gives

$$\hat{\sigma}_{12}{}^2 = \frac{466(11.76)^2 + 203(9.410)^2}{466 + 203} \cdot \frac{466 + 203}{(466)(203)}$$

$$= 0.87129$$

Hence $\hat{\sigma}_{12} = 0.9334$

Then $\mu_1 - \mu_2 = \bar{x}_1 - \bar{x}_2 - t\hat{\sigma}_{12}$

$$= 60.78 - 56.49 \pm 2.576(0.9334)$$

$$= 4.29 \pm 2.40$$

$$= 1.89 \text{ or } 6.69$$

Since these limits do not contain zero, the difference of the means is significant at the 1 per cent level.

### 7. The population standard deviation for the 1959 Ability Test scores.

| Class Interval | Class Mark x | Frequency f | u | fu | fu(u) = $fu^2$ | $f(u + 1)^2$ |
|---|---|---|---|---|---|---|
| 21-25 | 23 | 1 | -10 | -10 | 100 | 81 |
| 26-30 | 28 | 7 | -9 | -63 | 567 | 448 |
| 31-35 | 33 | 12 | -8 | -96 | 768 | 588 |
| 36-40 | 38 | 43 | -7 | -301 | 2107 | 1548 |
| 41-45 | 43 | 55 | -6 | -330 | 1980 | 1375 |
| 46-50 | 48 | 88 | -5 | -440 | 2200 | 1408 |
| 51-55 | 53 | 135 | -4 | -540 | 2160 | 1215 |
| 56-60 | 58 | 179 | -3 | -537 | 1611 | 716 |
| 61-65 | 63 | 190 | -2 | -380 | 760 | 190 |
| 66-70 | 68 | 224 | -1 | -224 | 224 | 0 |
| 71-75 | 73 | 286 | 0 | 0 | 0 | 286 |
| 76-80 | 78 | 249 | 1 | 249 | 249 | 996 |
| 81-85 | 83 | 278 | 2 | 556 | 1112 | 2502 |
| 86-90 | 88 | 285 | 3 | 855 | 2565 | 4560 |
| 91-95 | 93 | 212 | 4 | 848 | 3392 | 5300 |
| 96-100 | 98 | 163 | 5 | 815 | 4075 | 5868 |
| 101-105 | 103 | 89 | 6 | 534 | 3204 | 4361 |
| 106-110 | 108 | 16 | 7 | 112 | 784 | 1024 |
|  |  | 2512 |  | 1048 | 27858 | 32466 |
|  |  |  |  | X2 |  |  |
|  |  |  |  | 2096 |  |  |

Charlier check: $\sum f(u+1)^2 = \sum fu^2 + 2\sum fu + \sum f$

Left hand side = 32466

Right hand side = 27858 + 2096 + 2512 = 32466.

When using population statistics, the notation of formula (8) becomes

$$M\sigma_u^{\,2} = \sum f_i x_i^{\,2} - \frac{1}{M}\left(\sum f_i x_i\right)^2,$$

where M is the population size, o is the population standard deviation. Substituting appropriately gives

$$2512\sigma_u^{\,2} = 27858 - \frac{1}{2512}(1048)^2$$

$$= 27420.78$$

Then $\sigma_u^{\,2} = 27420.78/2512 = 10.9159$

$$\sigma_u = (10.9159)^{1/2} = 3.304$$

Finally, $\sigma_x = c\sigma_u \quad 5(3.304) = 16.52$

The standard deviation is 16.52.

### 8. The sample standard deviation for the 1959 Ability Test scores.

| Class Interval | Class Mark x | Frequency f | u | fu | fu(u) = $fu^2$ | $f(u + 1)^2$ |
|---|---|---|---|---|---|---|
| 21-25 | 23 | 1 | -10 | -10 | 100 | 81 |
| 26-30 | 28 | 1 | -9 | -9 | 81 | 64 |
| 31-35 | 33 | 5 | -8 | -40 | 320 | 245 |
| 36-40 | 38 | 10 | -7 | -70 | 490 | 360 |
| 41-45 | 43 | 9 | -6 | -54 | 324 | 225 |
| 46-50 | 48 | 20 | -5 | -100 | 500 | 320 |
| 51-55 | 53 | 19 | -4 | -76 | 304 | 171 |
| 56-60 | 58 | 42 | -3 | -126 | 378 | 168 |
| 61-65 | 63 | 33 | -2 | -66 | 132 | 33 |
| 66-70 | 68 | 46 | -1 | -46 | 46 | 0 |
| 71-75 | 73 | 51 | 0 | 0 | 0 | 51 |
| 76-80 | 78 | 41 | 1 | 41 | 41 | 164 |
| 81-85 | 83 | 62 | 2 | 124 | 248 | 558 |
| 86-90 | 88 | 46 | 3 | 138 | 414 | 736 |
| 91-95 | 93 | 47 | 4 | 188 | 752 | 1175 |
| 96-100 | 98 | 38 | 5 | 190 | 950 | 1368 |
| 101-105 | 103 | 31 | 6 | 186 | 1116 | 1519 |
| 106-110 | 108 | 3 | 7 | 21 | 147 | 192 |
|  |  | 505 |  | 291 | 6343 | 7430 |
|  |  |  |  | X2 |  |  |
|  |  |  |  | 582 |  |  |

Charlier check: $\sum f(u + 1)^2 = \sum fu^2 + 2\sum fu + \sum f$

Left hand side = 7430

Right hand side = 6343 + 582 + 505 = 7430

Using formula (8),

$Ns_u^2 = \sum f_i x_i^2 - \frac{1}{N}(\sum f_i x_i)^2$, and substituting appropriately,

$$505s_u^2 = 6343 - \frac{1}{505}(291)^2$$

$$= 6175.31$$

Then $s_u^2 = 6175.31/505 \quad 12.228$

$$s_u = (12.228)^{1/2} = 3.49$$

Finally, $s_x = cs_u = 5(3.49) = 17.45.$

The standard deviation is 17.45.

9. Confidence limits for the population mean, 1959 Ability Test.

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{N}} \qquad (1)$$

$$= 75.7 \pm \frac{2.576(17.45)}{\sqrt{505}}$$

$$= 75.7 \pm 2.00$$

$$= 73.7 \text{ or } 77.7.$$

The true mean for the population was found to be 75.0, which is between these limits. Thus the means of the sample and the population do not differ significantly at the 1 per cent level.

### 10. Confidence limits for the population standard deviation, 1959 Ability Test.

Using the method described on page 157, the standard error of estimate is $17.45/\sqrt{1010}$, which is equal to 0.55. Two standard errors are therefore equal to 1.10.

There is a probability of 95 per cent that the population standard deviation does not differ from the sample standard deviation by more than 1.10 each way; in other words, that it lies between 16.35 and 18.55. The true population standard deviation was found to be 16.52. Thus the standard deviations of the population and the sample are not significantly different at the 5 per cent level of confidence.

B29788